



Potsdam  
THE STATE UNIVERSITY OF NEW YORK

# SOAR: The Sky in Motion Life on the Tilted Teacup Ride

The Day in all its Glory:  
The Analemma

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ST. LAWRENCE UNIVERSITY

# Kiva

December 1997 - October 27, 2009



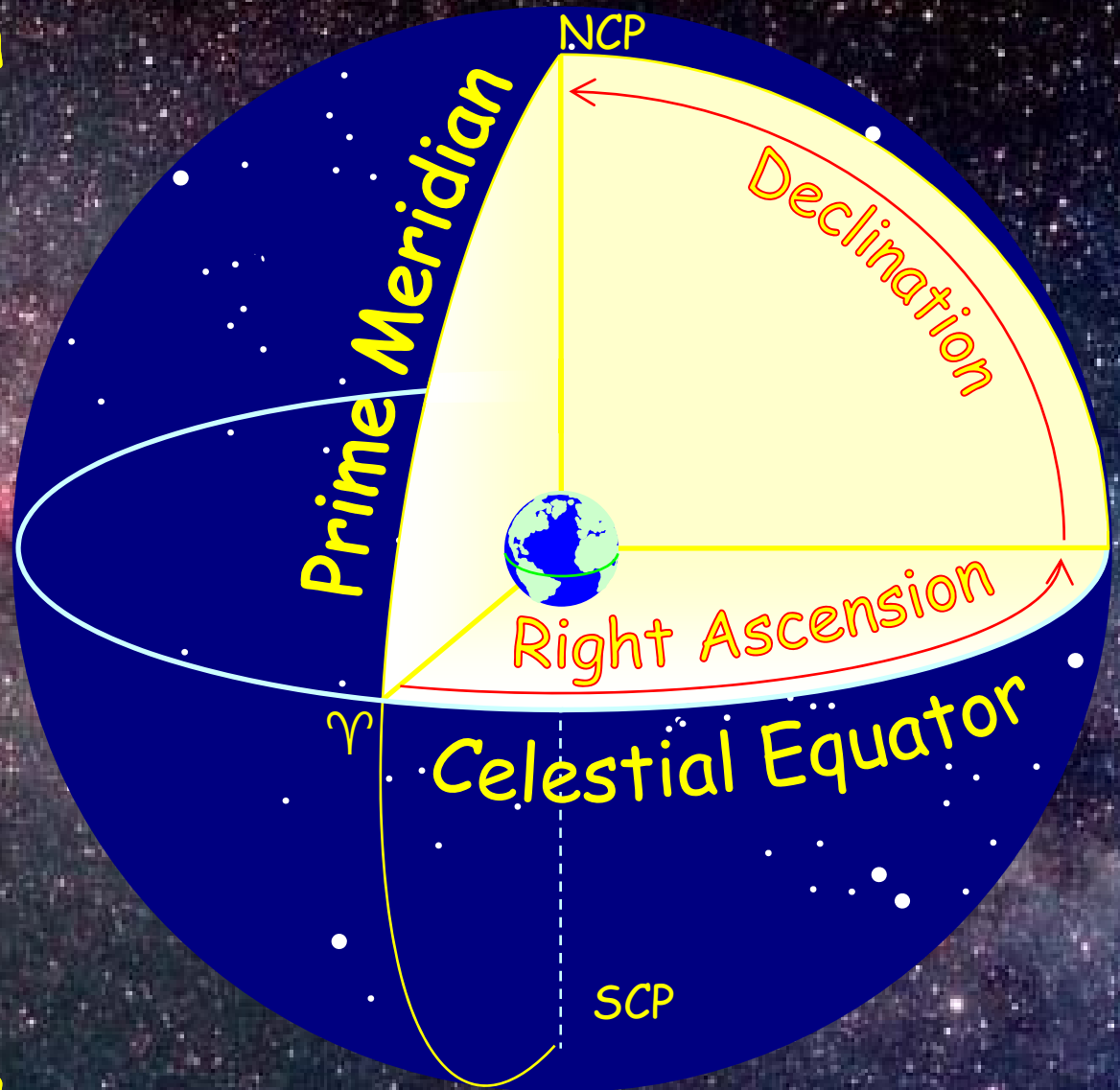
# Celestial Coordinates

## ☆ Right Ascension

- 🌐 RA or  $\alpha$
- 🌐 From prime meridian ( $0^h$ ) to  $23^h59^m59^s$  Eastward

## ☆ Declination

- 🌐 Dec or  $\delta$
- 🌐 From celestial equator ( $0^\circ$ ) to poles N & S  $90^\circ$

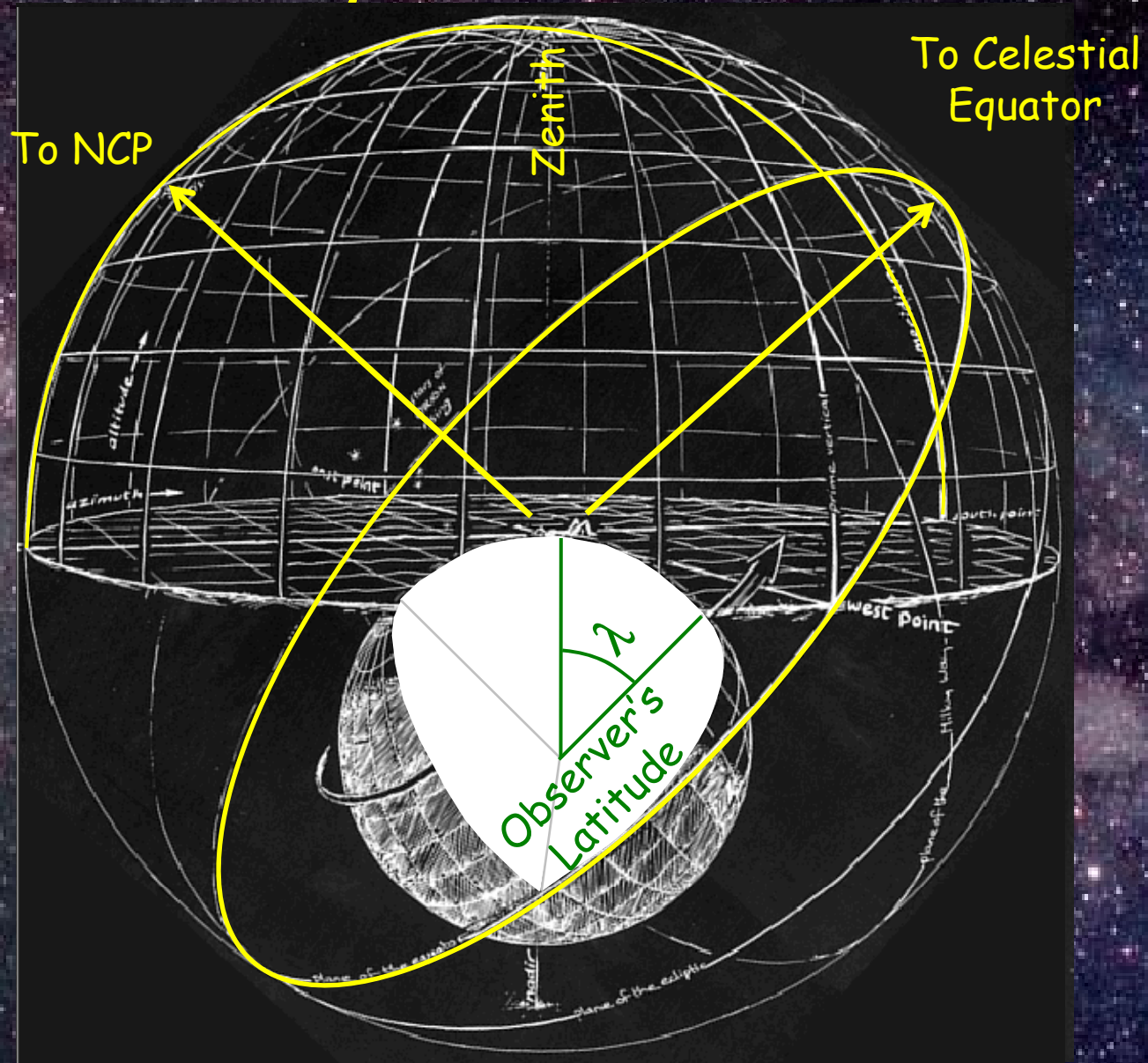


# Tilted Sky

☆ Observers see sky "tilted" due to latitude

We see ourselves "on top" of the Earth, beneath the sky.

So we see sky motions tilted



# Standard Clock Time

☆ Every Longitude at different time

It's 6 pm  
(sunset).

It's 3 pm.

It's 9 pm.

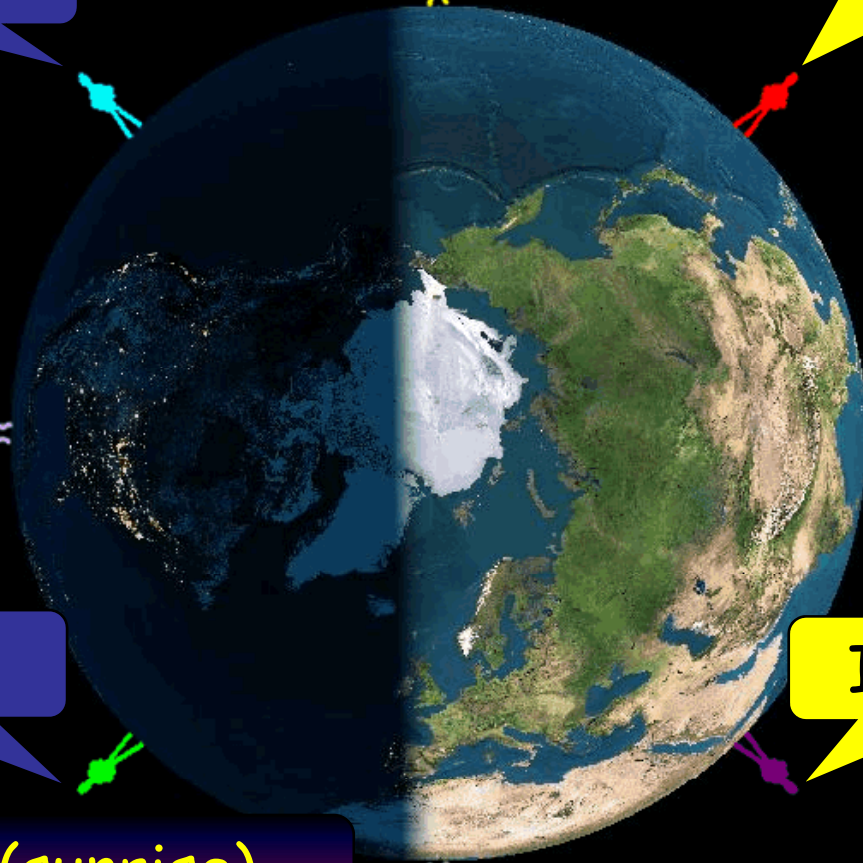
It's  
midnight.

It's  
noon.

It's 3 am.

It's 9 am.

It's 6 am (sunrise).



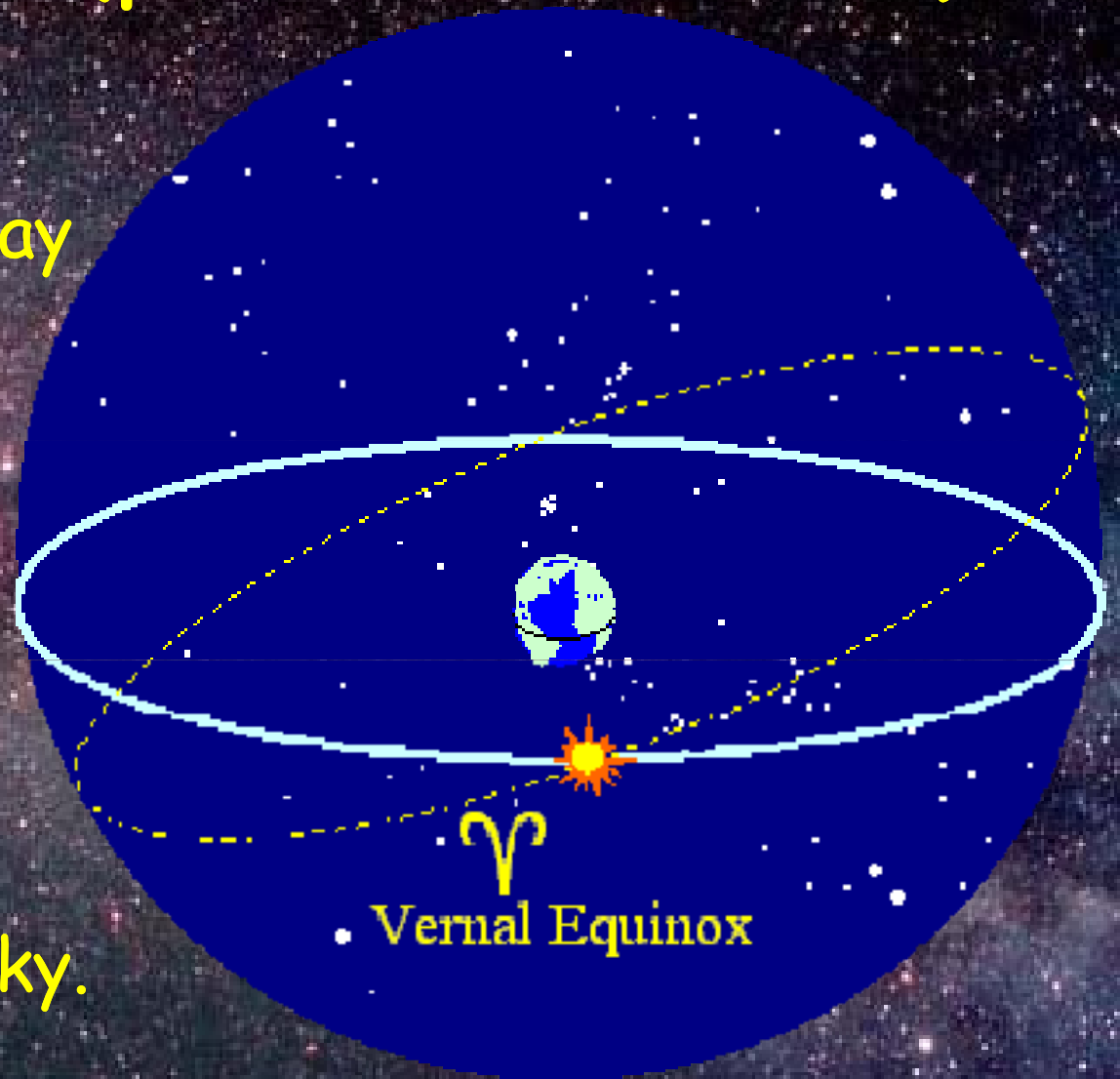
# The Ecliptic (path of the sun)

## ☆ View from Earth

🌍 Sun moves  $\sim 1^\circ$ /day eastward across stars

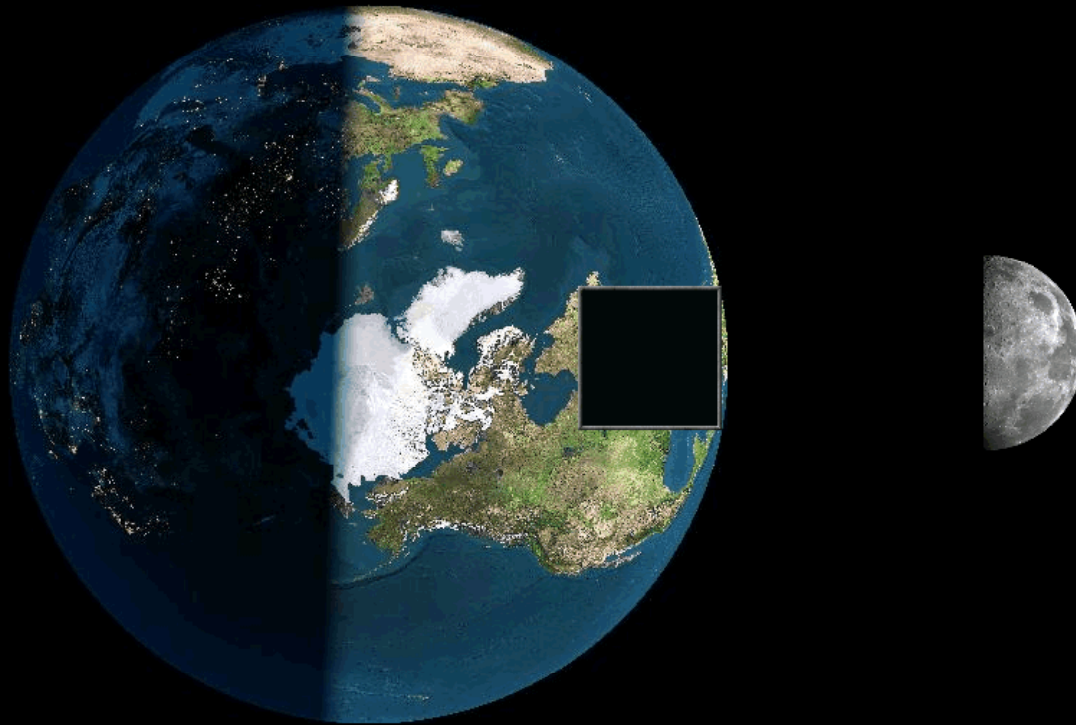
🌍 Sun moves north and south in declination

🌍 Solstices & Equinoxes are positions in the sky.



This motion is through the YEAR!

Moon  
Phase is  
lit moon  
visible



Insert is  
moon as  
see from  
Earth

# Moon Phases

☆ **New Moon: Elongation =  $0^\circ$**  (angle from sun to moon)

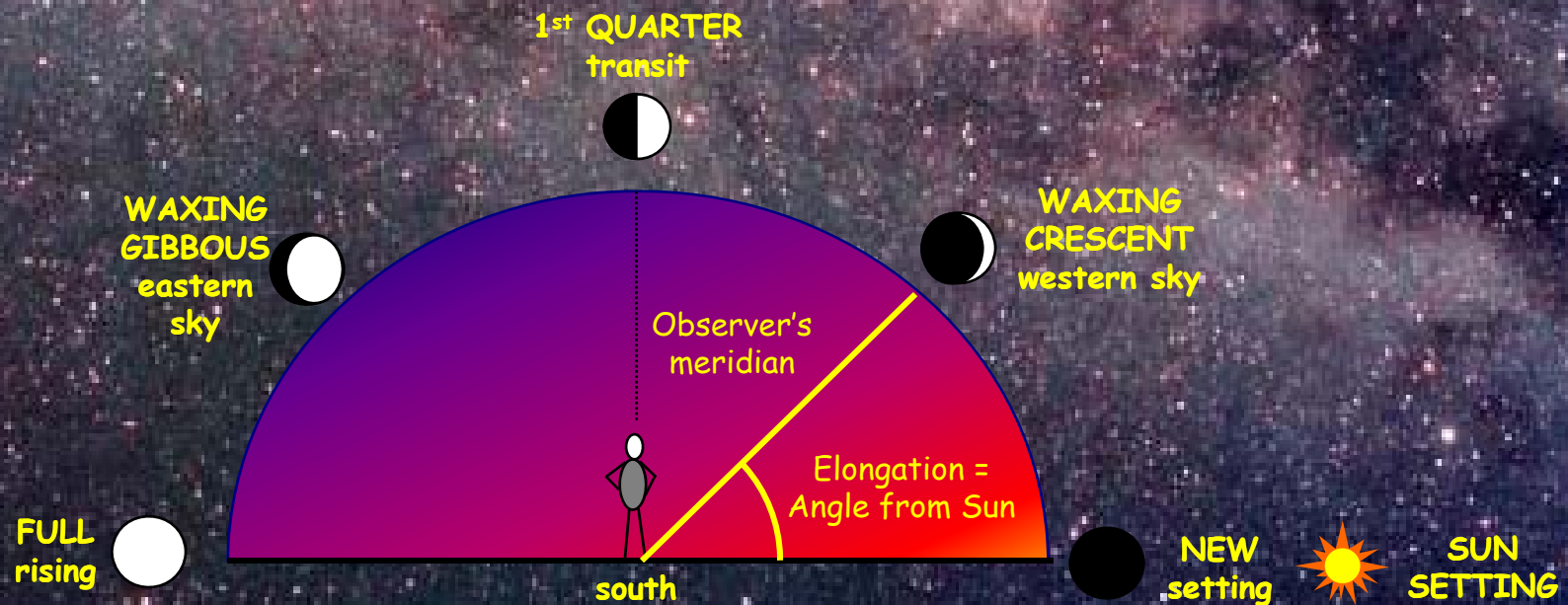
🌍 **Waxing Phases: visible after sunset**

› Waxing Crescent:  $0^\circ < \text{Elongation} < 90^\circ \text{ E}$

› First Quarter: Elongation =  $90^\circ \text{ E}$

› Waxing Gibbous:  $90^\circ \text{ E} < \text{Elongation} < 180^\circ$

☆ **Full Moon: Elongation =  $180^\circ$**



# Moon Phases

☆ Full Moon: Elongation =  $180^\circ$

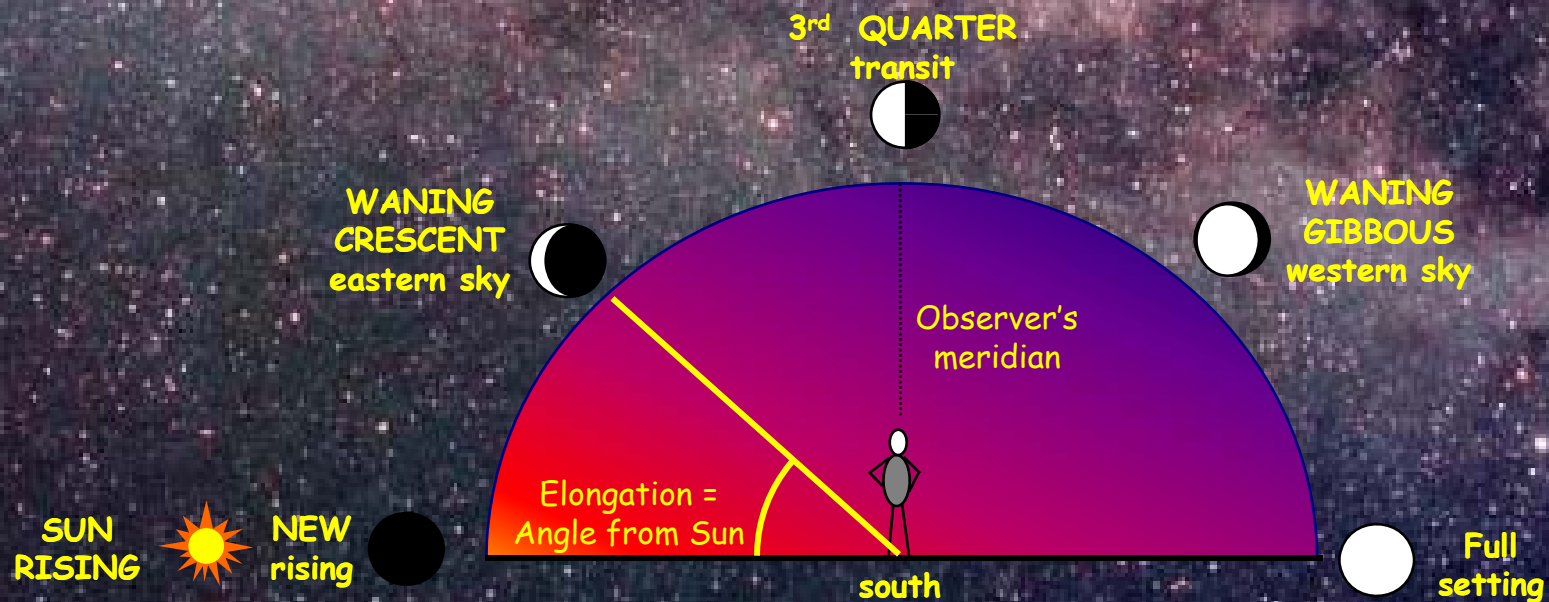
🌍 Waning Phases: visible before sunrise

› Waning Gibbous:  $90^\circ \text{ W} < \text{Elongation} < 180^\circ$

› Third Quarter: Elongation =  $90^\circ \text{ W}$

› Waning Crescent:  $0^\circ < \text{Elongation} < 90^\circ \text{ W}$

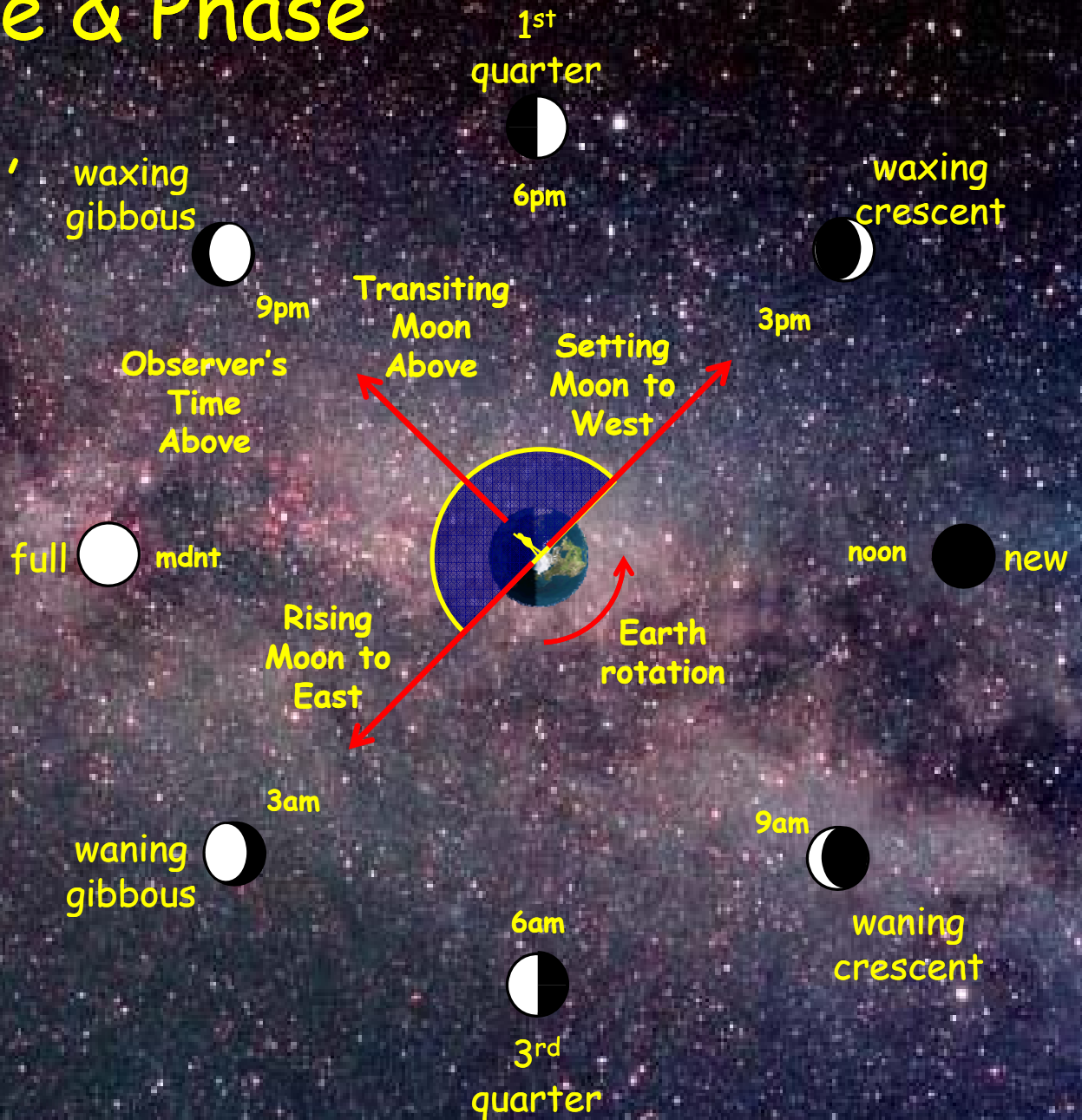
☆ New Moon: Elongation =  $0^\circ$



# Fun with Time & Phase

Determine rising, transit and setting times of each phase

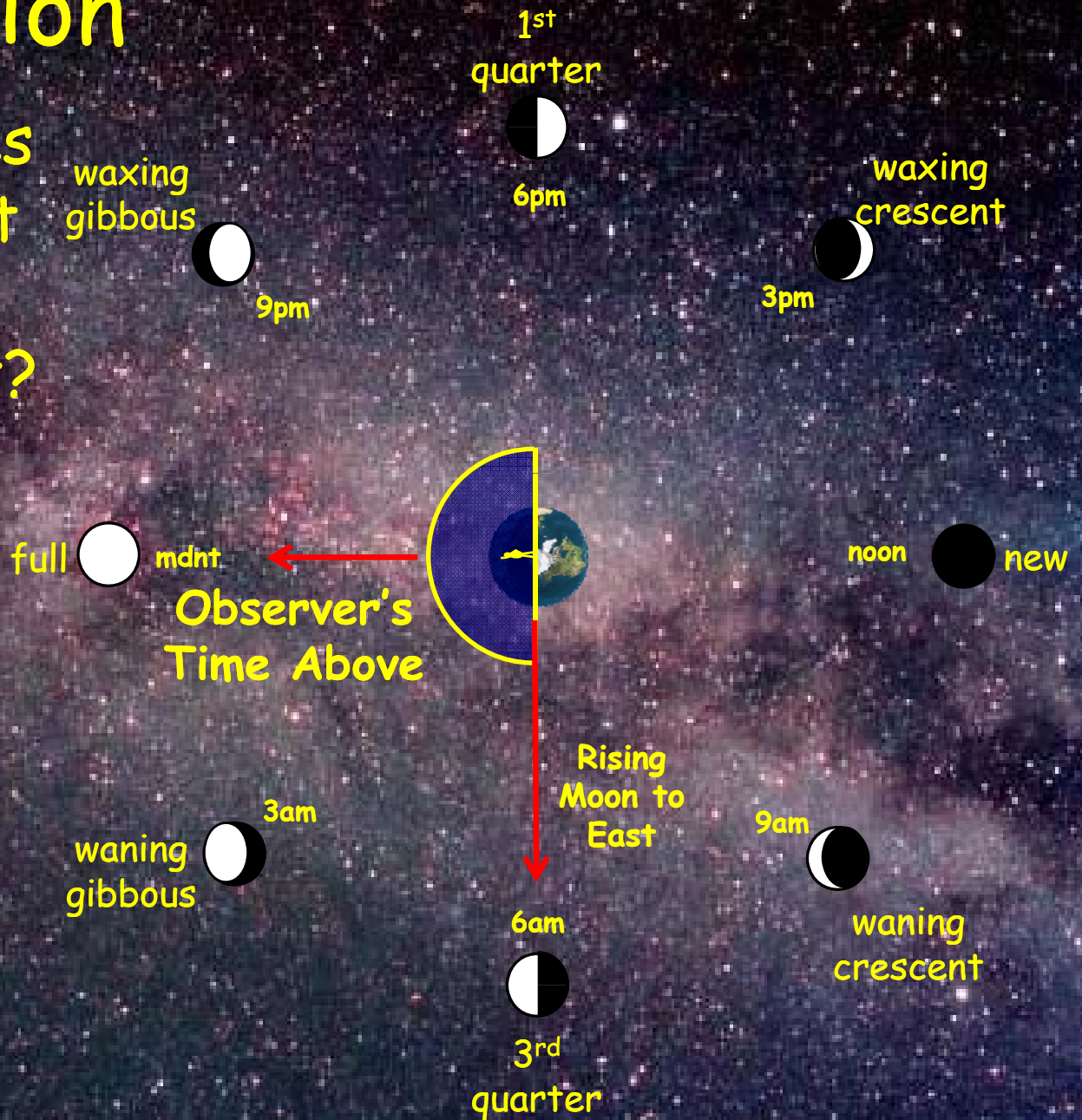
Time is the one above the observer's head!



# Question

An observer sees the moon rise at midnight.  
What phase is it?

- a) First Quarter
- b) Full Moon
- c) 3<sup>rd</sup> Quarter



# Time

## ☆ Clock Time

- 🌍 the position of the mean sun at TZ center
  - › eg. 12 pm = transit of mean sun (avg. of analemma)
- 🌍 Mean Solar Day = 24:00:00 (hours:min:sec of time)

## ☆ Solar Time

- 🌍 the position of the sun wrt the observer
  - eg. Noon = sun transits
- 🌍 Solar Day varies as shown by analemma

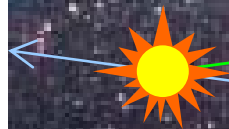
## ☆ Sidereal Time

- 🌍 the position of  $\Upsilon$  wrt the observer
  - › eg. 0<sup>h</sup> Local Sidereal Time (LST) =  $\Upsilon$  transits
  - › Sidereal time = R.A. on the meridian
- 🌍 Sidereal Day = 23:56:00

# The Sidereal Day

Sidereal Day: 360° rotation puts star back on meridian

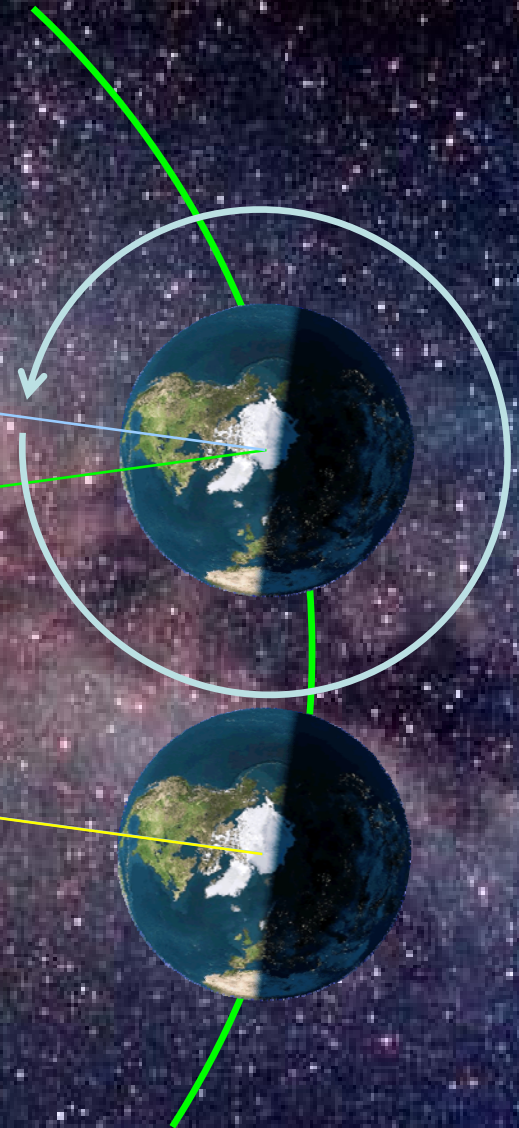
to distant star



~1° along orbit

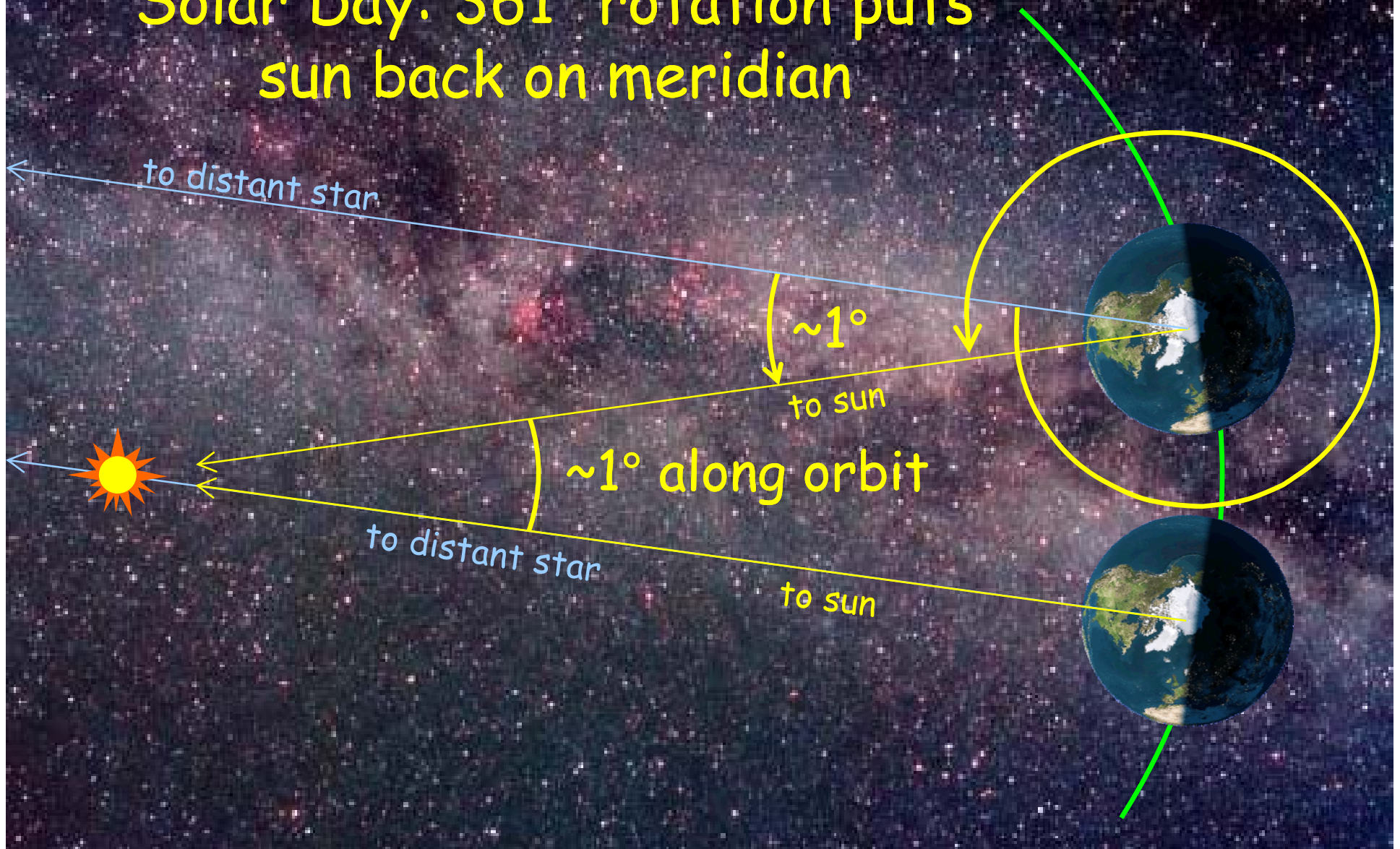
to distant star

to sun



# The Solar Day

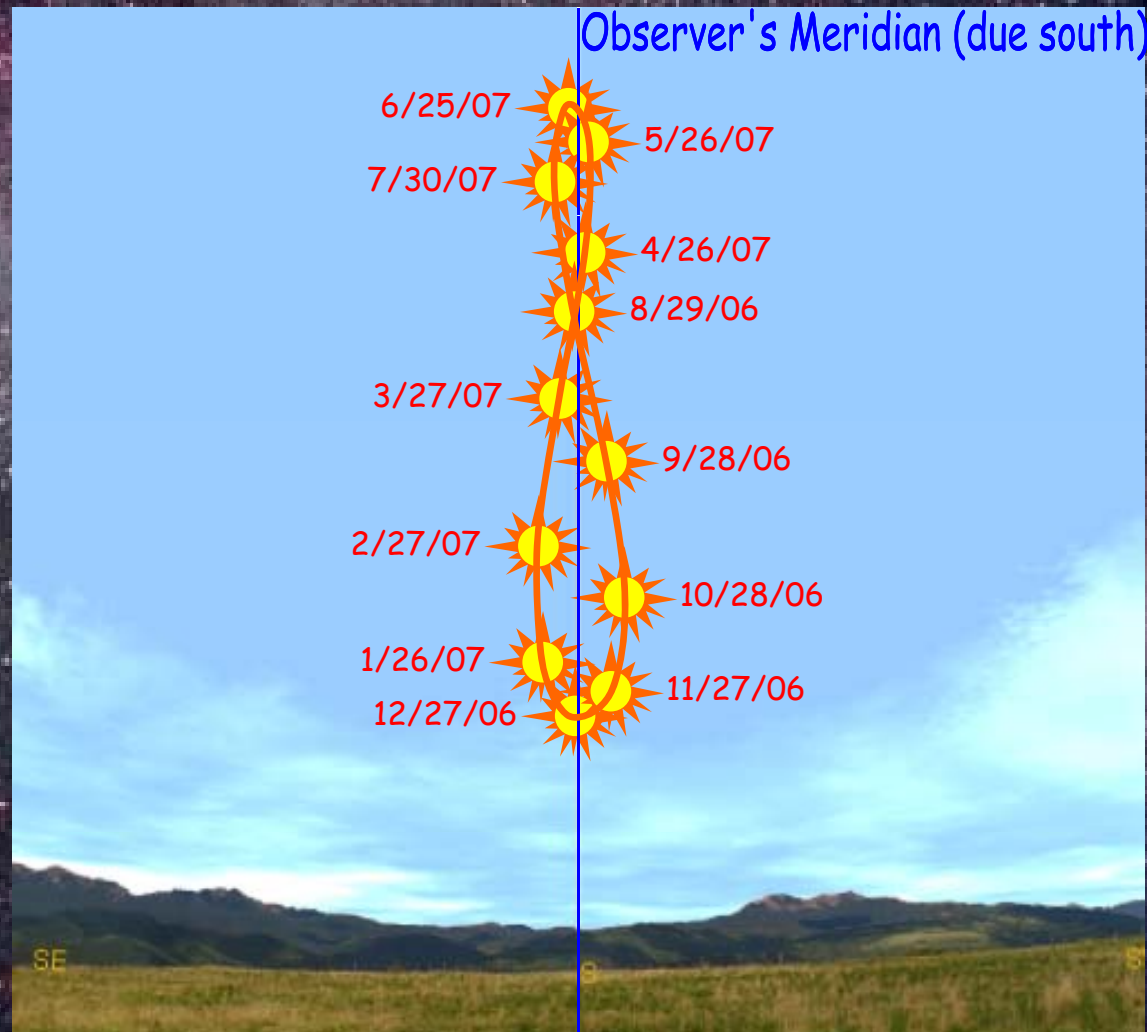
Solar Day:  $361^\circ$  rotation puts sun back on meridian



# The Sun at Noon

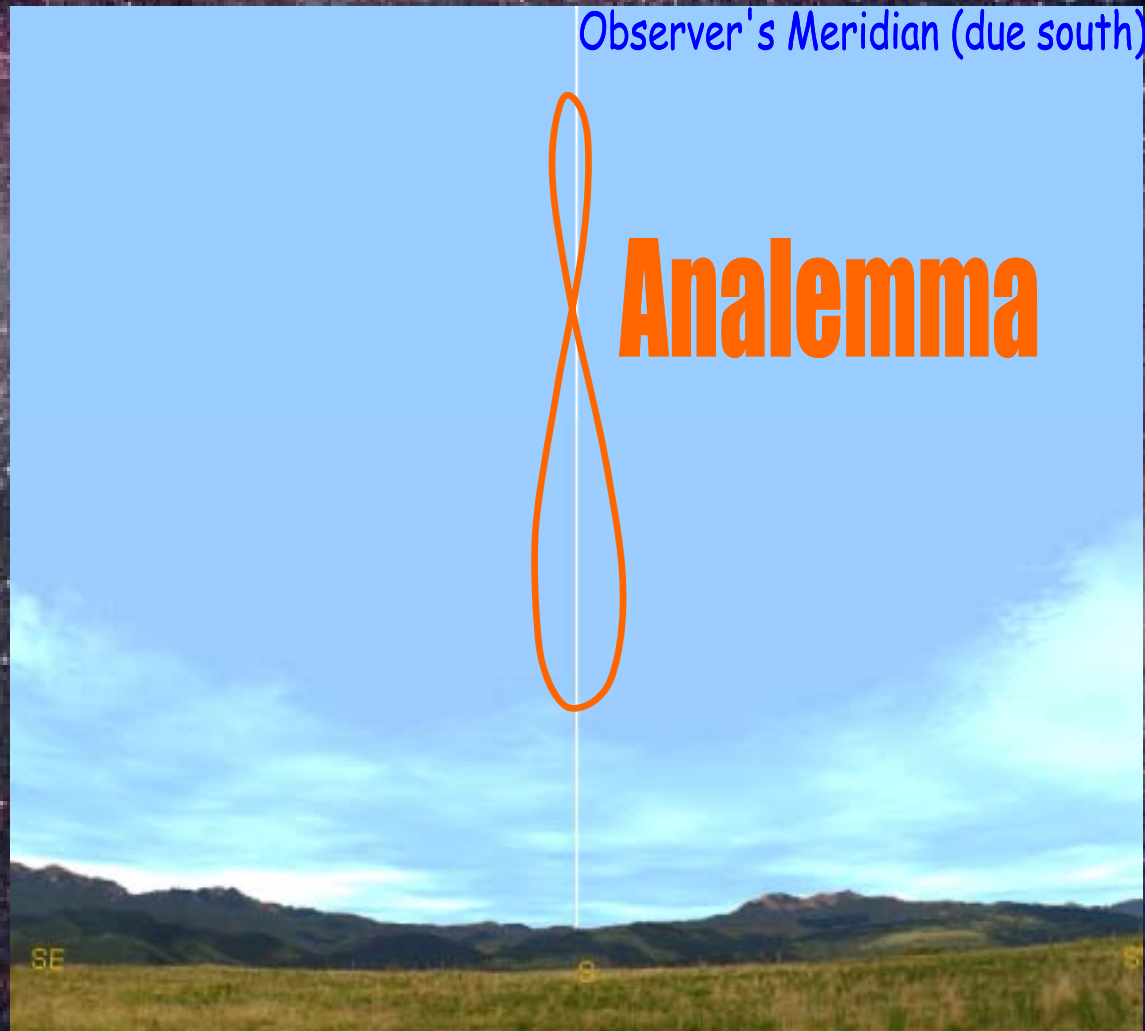
☆ Noon  $\Rightarrow$  Sun on meridian

☆ Sun's position varies: the Analemma



# The Sun at Noon

- ☆ Noon  $\Rightarrow$  Sun on meridian
- ☆ Sun's position varies: the Analemma



# The Analemma

## ☆ Position of true sun at clock noon

### 🌍 Clock Noon

- › 12:00 pm in a 24:00:00 day
- › Position of Mean Sun at noon

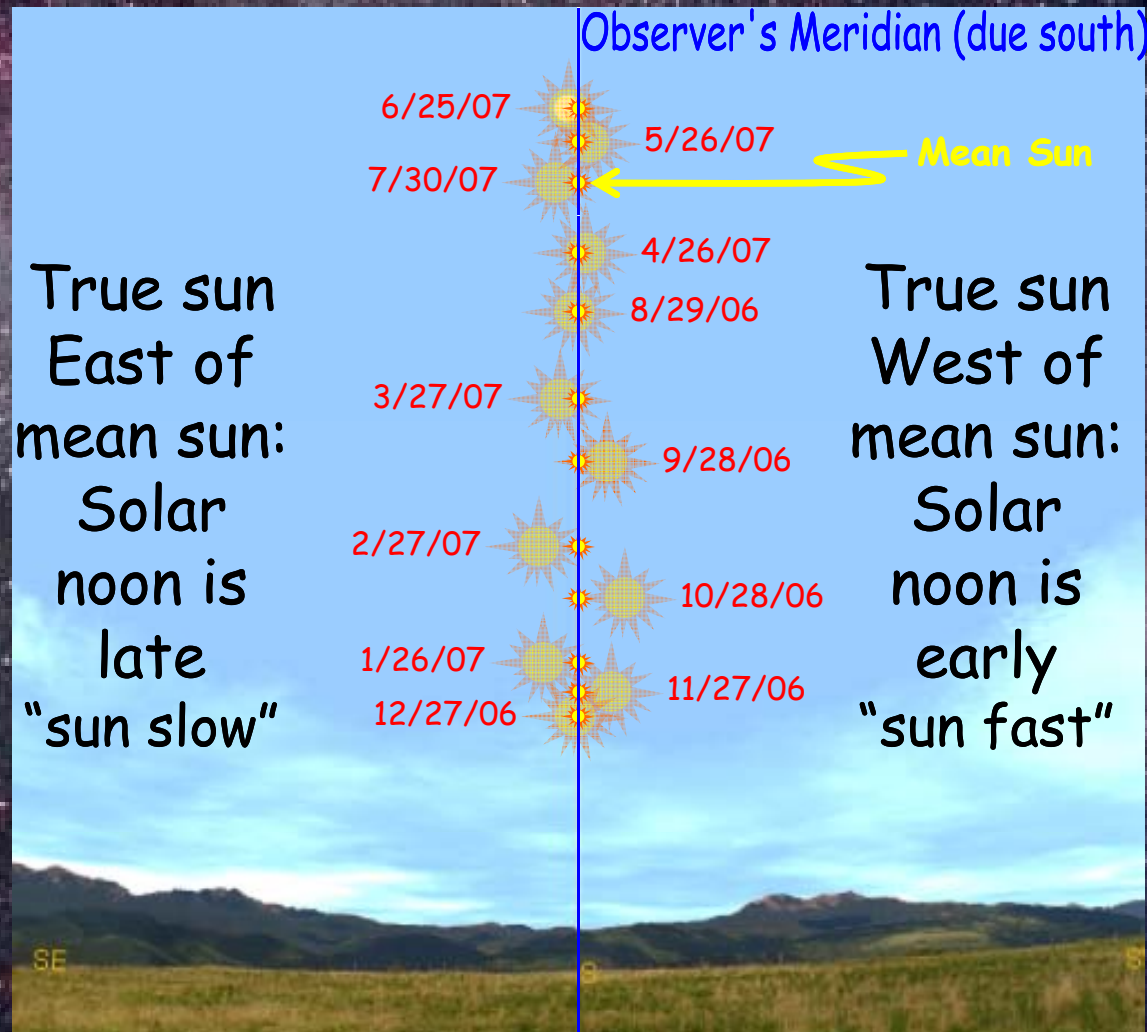
### 🌍 True Sun's Position

- › varies due to Sun's speed along path
- ↳ varies due to elliptical path
- ↳ varies due to tilted path



# Mean Sun & True Sun

- ☆ Mean sun on meridian defines clock noon
- ☆ True sun on meridian defines solar noon



# The Calendar

## ☆ The Year

- 🌍 Tropical (equinox to equinox) = 365.242190 d
- 🌍 Sidereal (star to star) = 365.256363 d
- 🌍 Anomalistic (perigee to perigee) = 365.259635 d
- 🌍 Lunar (node to node) = 346.620076 d

☆ Julian Calendar (45 BCE) = 365.25 d

🌍 Add 1 day every four years

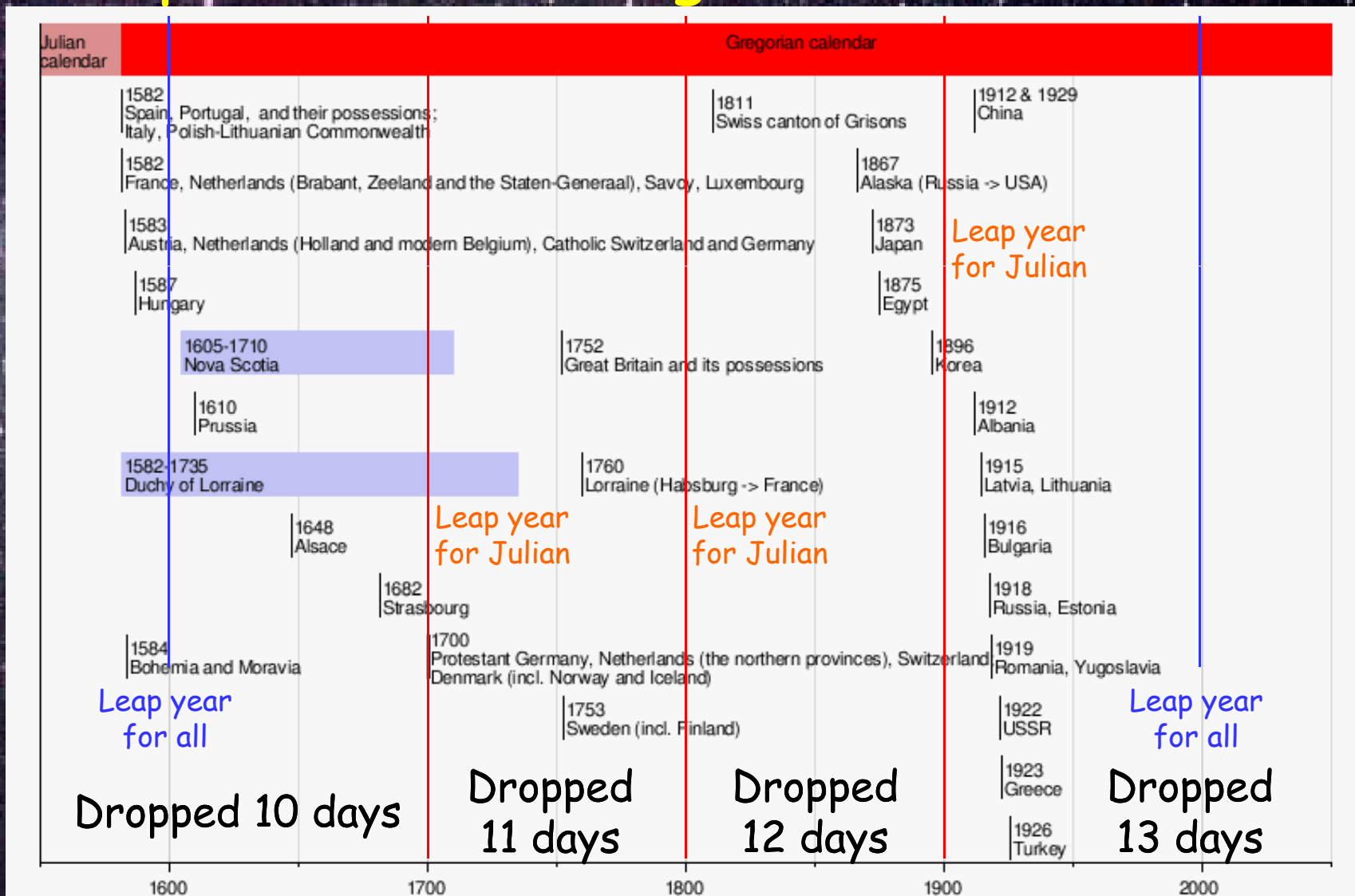
›  $365.25 - 365.24219 = 0.00781$  days/year too many

⇒ Extra day every 128 years



# Doing the Math

## ☆ Adoption of the Gregorian Calendar



# Doing the Math

☆ Old Style & New Style dates  
Marriage certificate from Warsaw (then in Russia)

Marriage  
Dated 3/16/1907

Certificate  
Dated Nov/Dec 23/6

160  
1907г

ГОРОДЪ ВАРШАВА. №  
Рим.-Кат. ПРИХОДЪ СВЯТАГО ЮАННА

Въ употребленіе рекрутскаго набора  
или книгъ народонаселенія

**СВИДѢТЕЛЬСТВО О БРАКОСОЧЕТАНІИ**  
ВЫДАННОЕ НА ОСНОВАНІИ МЕТРИЧЕСКИХЪ КНИГЪ

Объявляеть, что Сигизмундъ Францисекъ Шорновскій лѣтъ отъ роду 27  
съ Маргариетой Анной урожденною Шорновскій лѣтъ отъ роду 29

завключилъ }  
брачный союзъ }  
Дня 31  
мѣсяца Октябрия  
Тысяча двѣдѣсяти седьмано году 1907 года.

Достоверность че о симъ свидѣтельствую.


Г. Варшава Нояб./Дек. мѣсяца 23 / 6 дня 1907 года

Чимовникъ Гражданскаго Состоянія

Приставъ Исполнительной Полиціи 4 Участка Ашук  
Собственноручную подпись Ксендза Франциска Шорновскаго  
Самъ свидѣтельствуеть.

Г. Варшава Ноябрь 24 мѣсяца 24 1907 года

Je przyjmuję



# Doing the Math

## ☆ Mean Sun

🌍 Projection of sun onto Celestial Equator

› moves  $360^\circ$  in one year (365.242191 days)

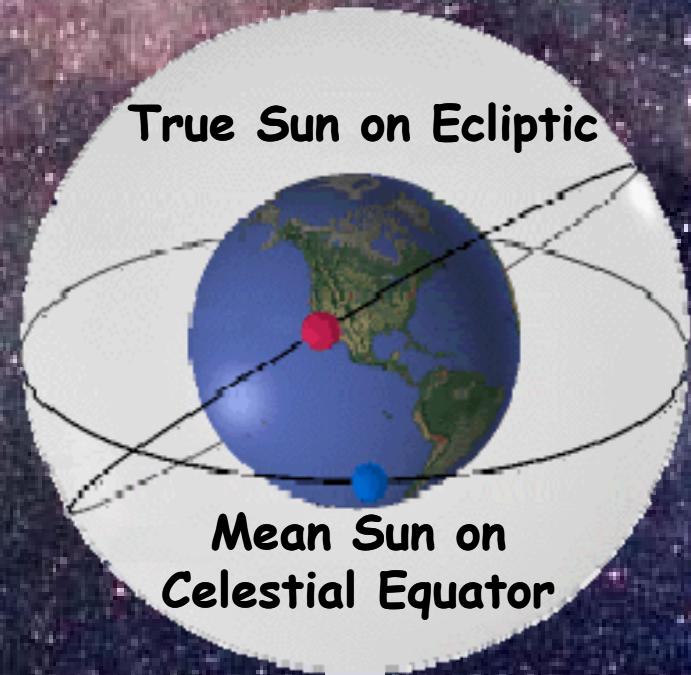
$$V_{\text{Mean Sun}} = \frac{360^\circ}{365.242191 \text{ days}} = 0.985647356^\circ/\text{day}$$

## ☆ True Sun

🌍 speed varies due to

› Sun's changing Declination

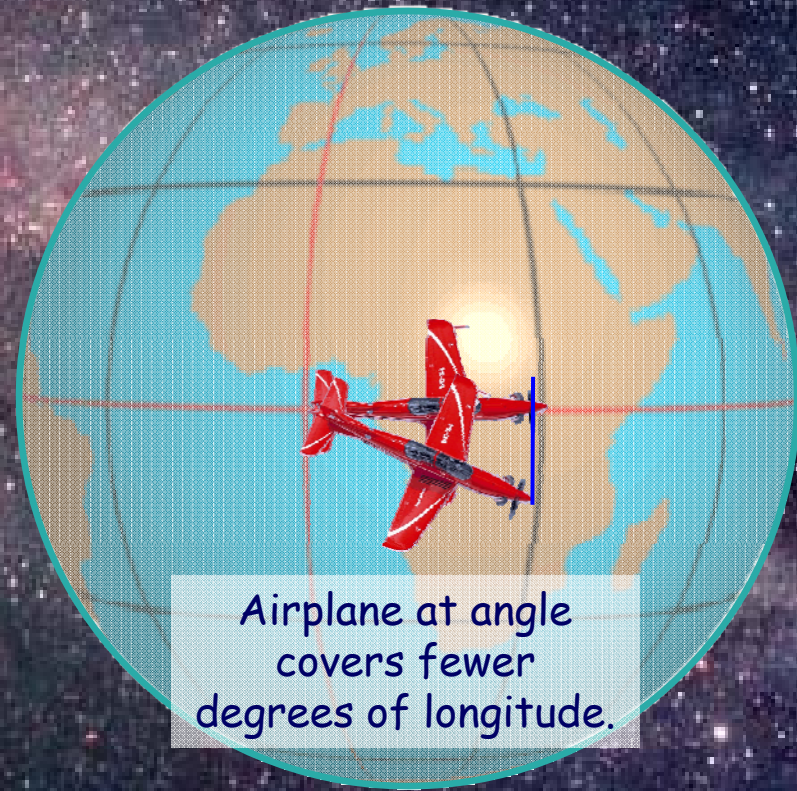
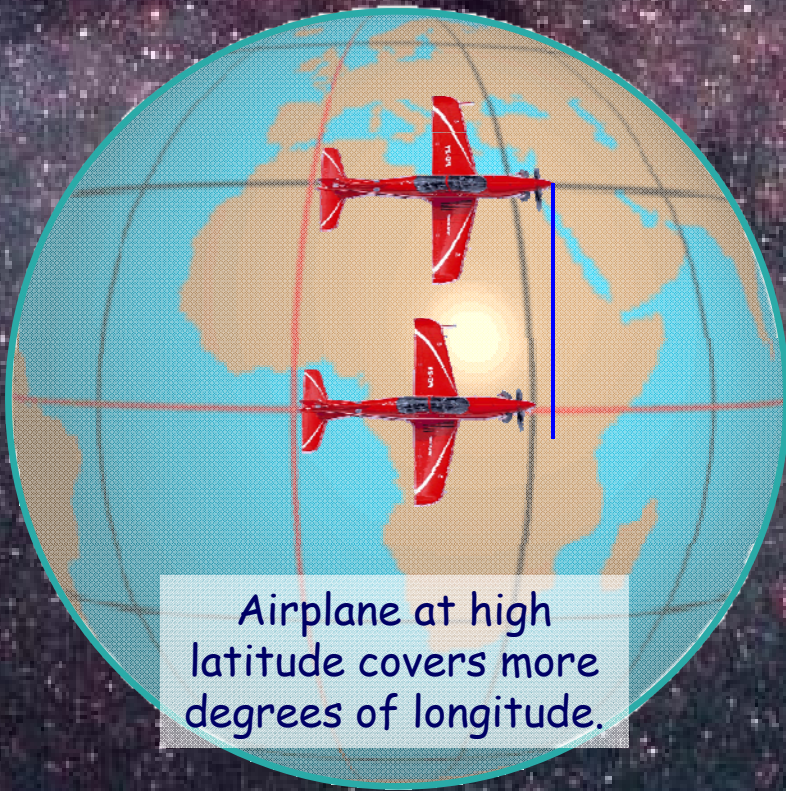
› Elliptical orbit



# Speed Variation Due to Tilt

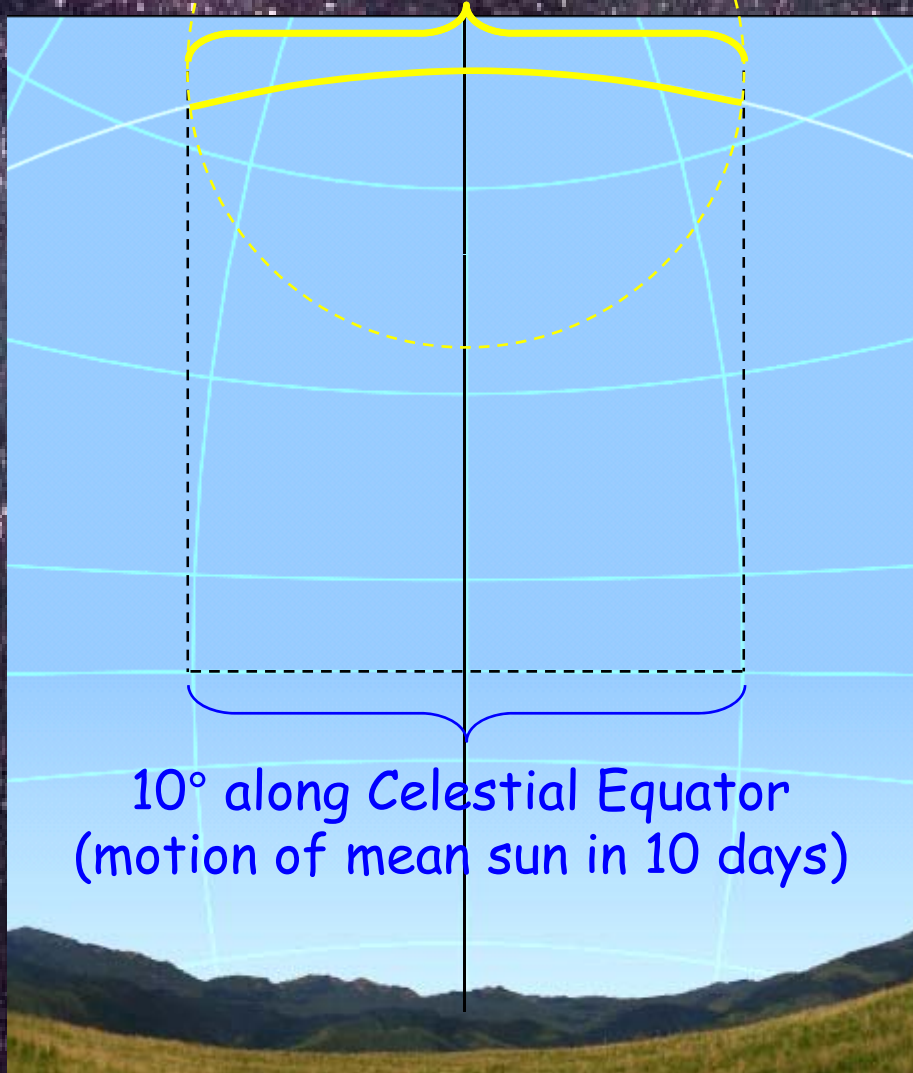
☆ Analogy: Airplanes on Earth

🌍 Both fly at same speed (mph)



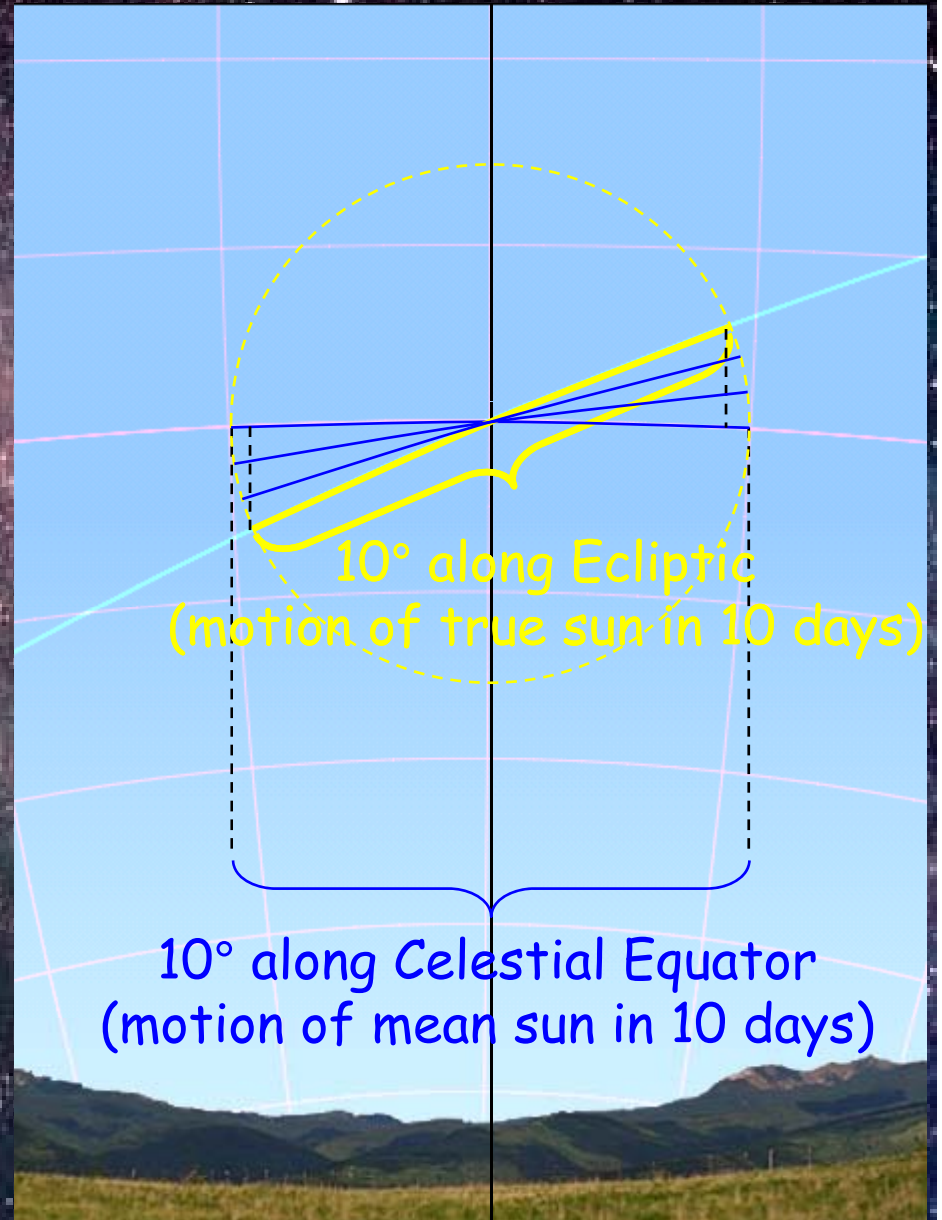
# Speed Variation Due to Tilt

10° along Ecliptic  
(motion of true sun in 10 days)



10° along Ecliptic  
(motion of true sun in 10 days)

10° along Celestial Equator  
(motion of mean sun in 10 days)



# Speed Variation Due to Tilt

10° along Ecliptic  
(motion of true sun in 10 days)

≈ 12° in right ascension  
(motion of true sun in sky)

At solstices,  
true sun moves > 1° each day  
⇒ true sun gets ahead of mean sun

10° along Celestial Equator  
(motion of mean sun in 10 days)

At equinoxes  
true sun moves < 1° each day  
⇒ true sun falls behind mean sun

10° along Ecliptic  
(motion of true sun in 10 days)

≈ 9° in right ascension  
(motion of true sun in sky)

10° along Celestial Equator  
(motion of mean sun in 10 days)

# True Sun Speed Variation

## ☆ Solstices

- 🌍 True sun and mean sun aligned, but ...
- 🌍 True sun getting ahead of mean at maximum rate

## ☆ Equinoxes

- 🌍 True sun and mean sun aligned, but
- 🌍 True sun getting behind mean at maximum rate

## ☆ Cross-Quarter Days

- 🌍 Between solstices & equinoxes
- 🌍 True sun farthest from mean
- 🌍 Switching between getting ahead & behind

# Cross Quarter Days

☆ Days  $\frac{1}{2}$  way between solstices & equinoxes

🌍 Beltane ~ May 1

›  $\frac{1}{2}$  way from Vernal Equinox to Summer Solstice

🌍 Lughnasa ~ August 2

›  $\frac{1}{2}$  way from Summer Solstice to Autumnal Equinox

🌍 All Hallows (Samhain) ~ November 1

›  $\frac{1}{2}$  way from Autumnal Equinox to Winter Solstice

🌍 Candlemas (Imbolc) ~ February 2

›  $\frac{1}{2}$  way from Winter Solstice to Vernal Equinox

If Candlemas Day be fair and bright,  
Winter will have another flight

If Candlemas Day be damp & black, It  
will carry cold winter away on its back.

# Speed Variation Due to Tilt

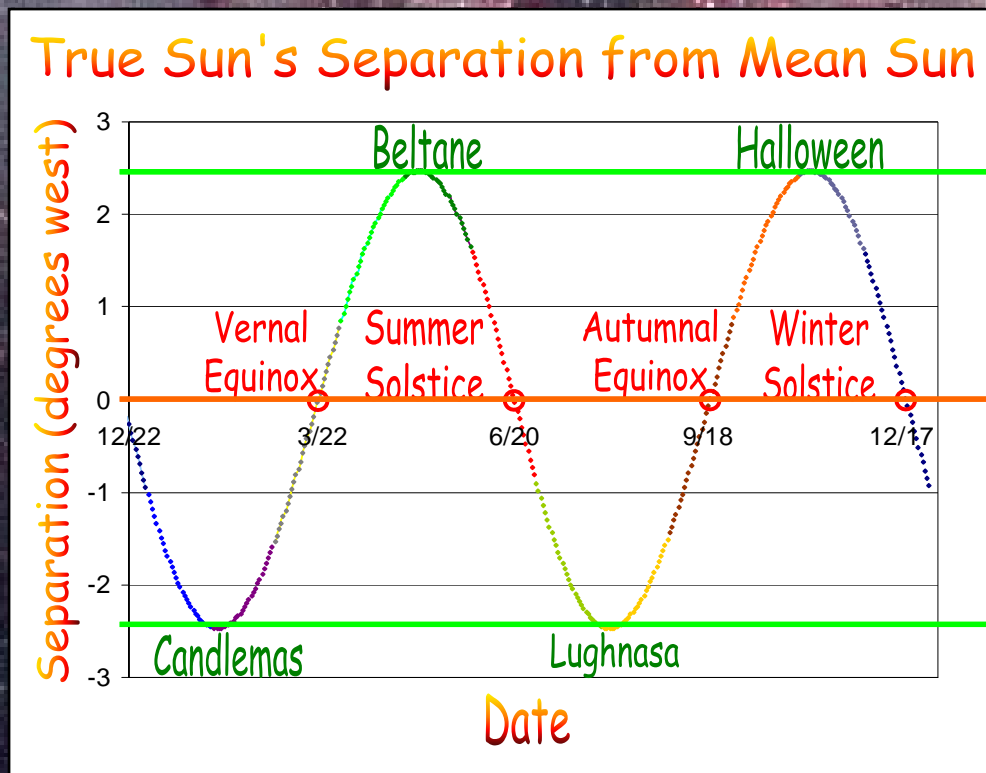
## ☆ Solstices & Equinoxes (June & December)

🌍 mean and true sun align, fastest rate of change

## ☆ Cross-Quarter Days

🌍 maximum separation of mean and true sun

🌍 switching direction of change



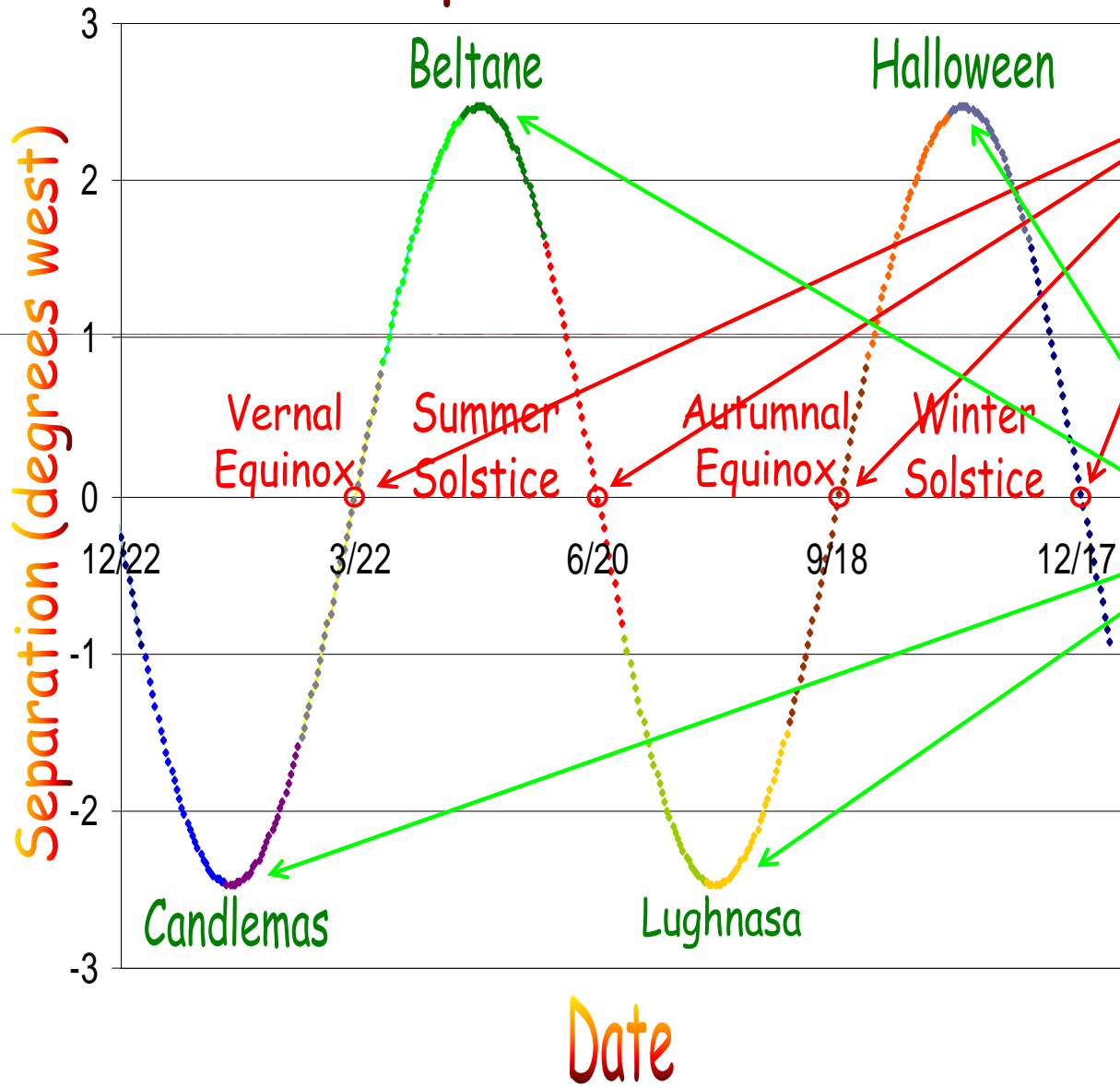
True sun  
farthest west  
(early, fast)

Mean sun  
& true sun  
aligned

True sun  
farthest east  
(late, slow)

# Speed Variation Due to Tilt

## True Sun's Separation from Mean Sun



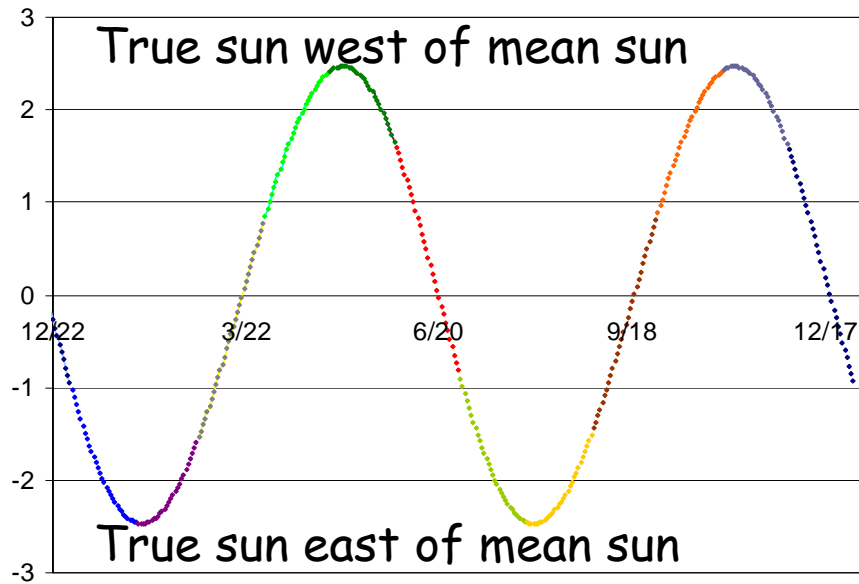
Maximum rate of change at solstices & equinoxes

Maximum difference at cross-quarter days (direction of change switches)

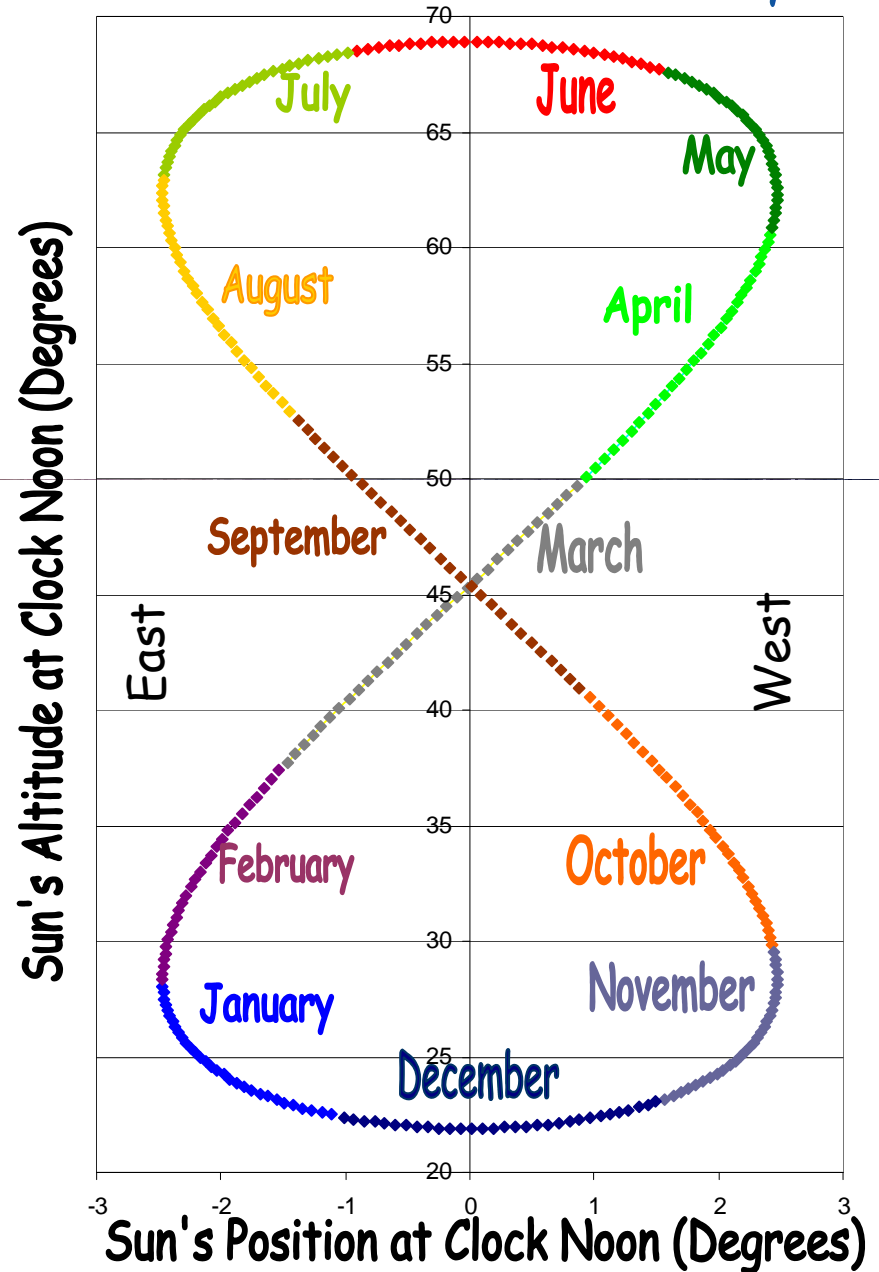
# Tilt Analemma

☆ Position of true sun through the year for Earth in a circular orbit.

Sun's Position at Clock Noon (Degrees)



# Tilt Analemma: Potsdam, NY



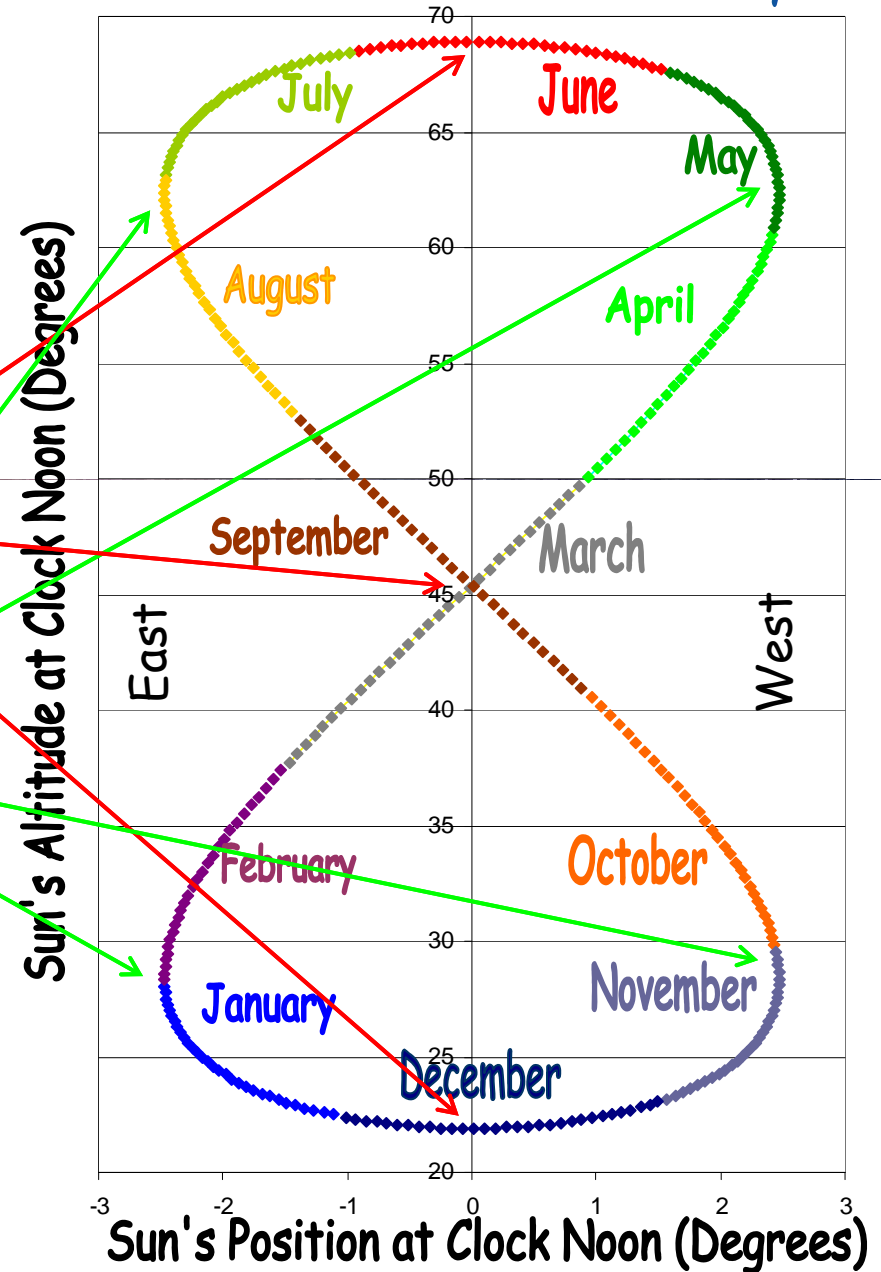
# Tilt Analemma

☆ Position of true sun through the year for Earth in a circular orbit.

True & mean sun aligned at solstices & equinoxes (rate of change maximum)

Maximum difference at cross-quarter days (direction of change switches)

# Tilt Analemma: Potsdam, NY



# Tilt Analemma

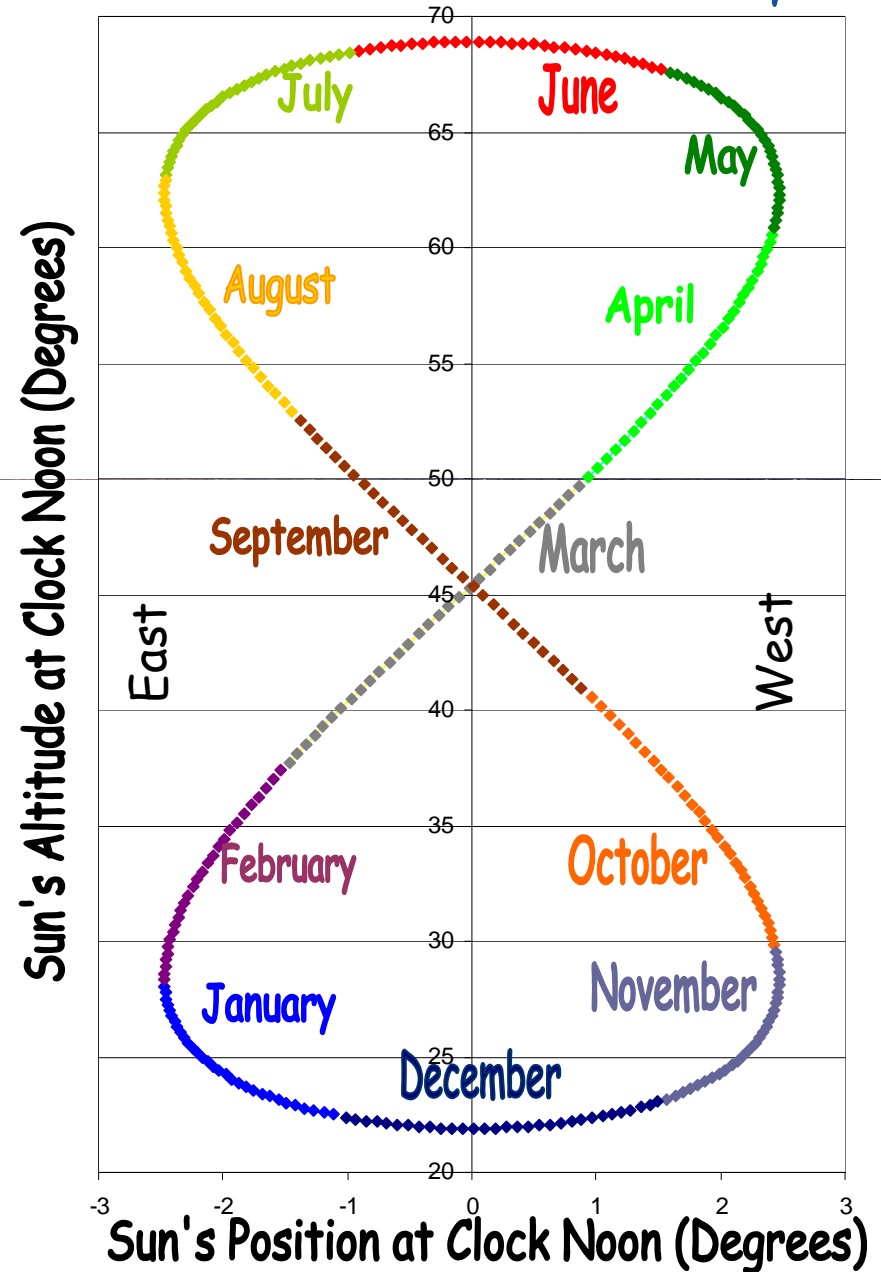
☆ Position of true sun through the year for Earth in a circular orbit.

True & mean sun aligned at solstices & equinoxes (rate of change maximum)

Maximum difference at cross-quarter days (direction of change switches)

But it's not this simple ...

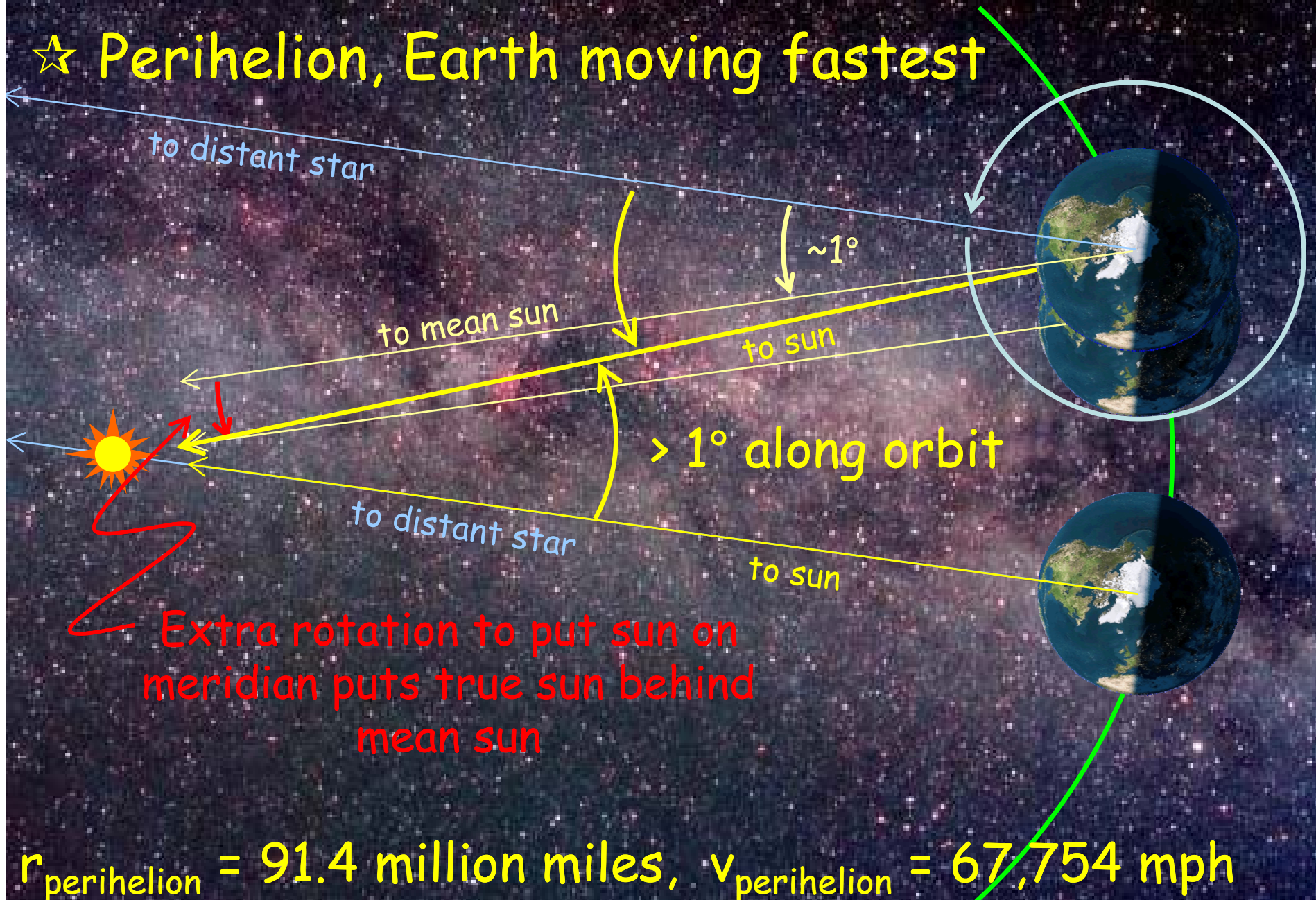
# Tilt Analemma: Potsdam, NY





# The Analemma

☆ Perihelion, Earth moving fastest

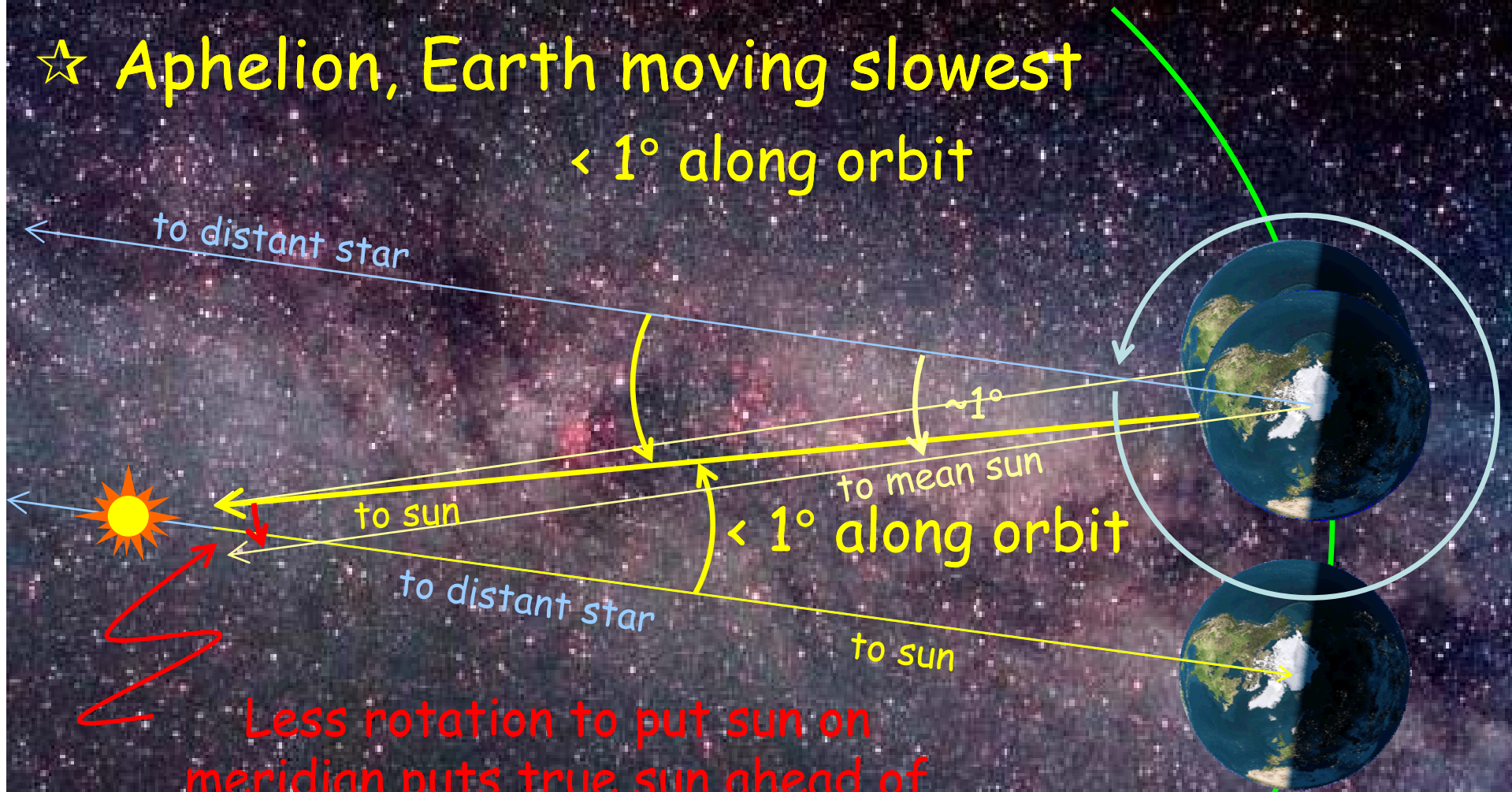


Extra rotation to put sun on meridian puts true sun behind mean sun

$r_{\text{perihelion}} = 91.4$  million miles,  $v_{\text{perihelion}} = 67,754$  mph

# The Analemma

☆ Aphelion, Earth moving slowest  
< 1° along orbit



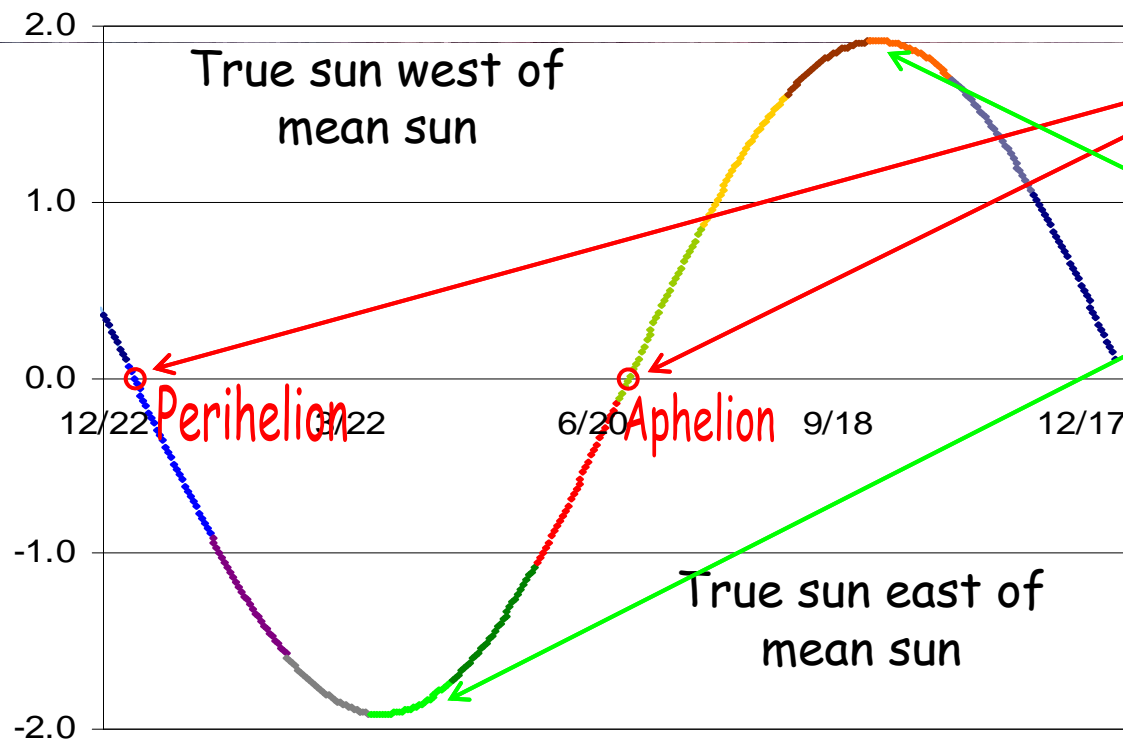
Less rotation to put sun on  
meridian puts true sun ahead of  
mean sun

$r_{\text{aphelion}} = 94.5 \text{ million miles}, v_{\text{aphelion}} = 65,527 \text{ mph}$

# Orbit Analemma

☆ Position of true sun through the year for Earth in an elliptical orbit just due to orbital speed variation

## Sun's Position at Clock Noon (Degrees)



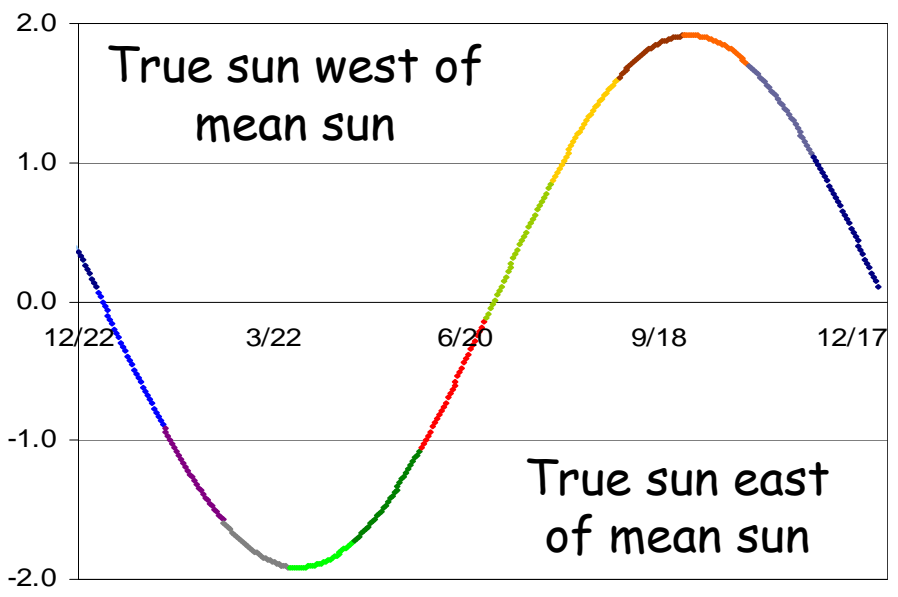
Maximum rate of change at perihelion & aphelion

Maximum difference at orbital mid-points (direction of change switches)

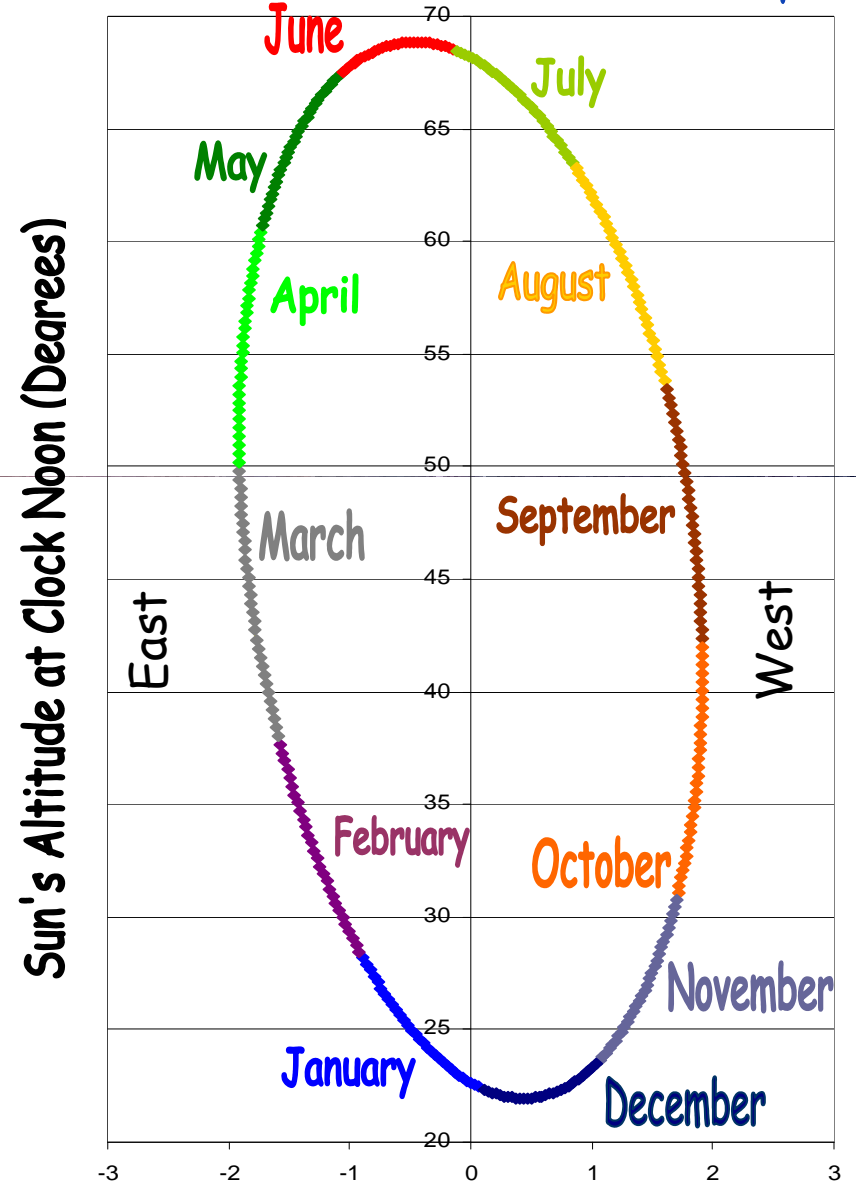
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Sun's Position at Clock Noon (Degrees)



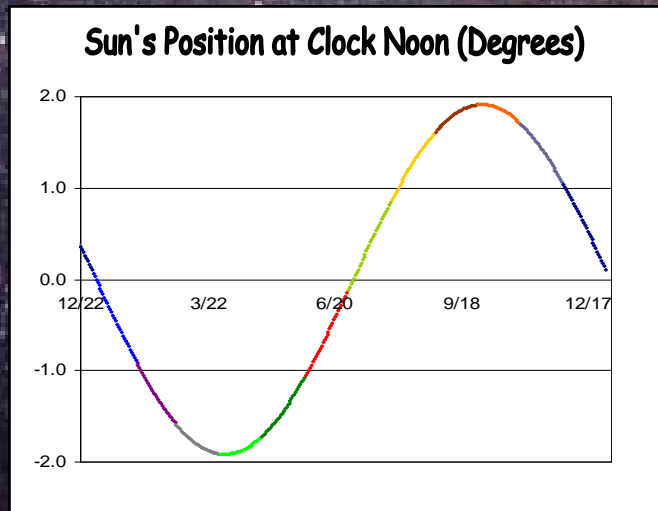
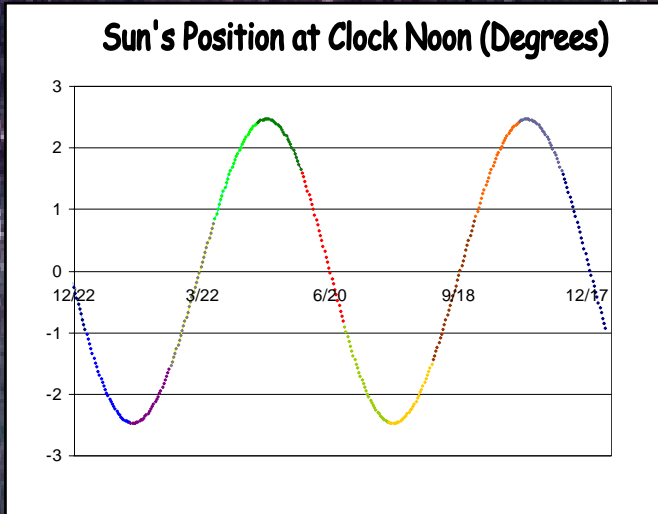
# Orbit Analemma: Potsdam, NY



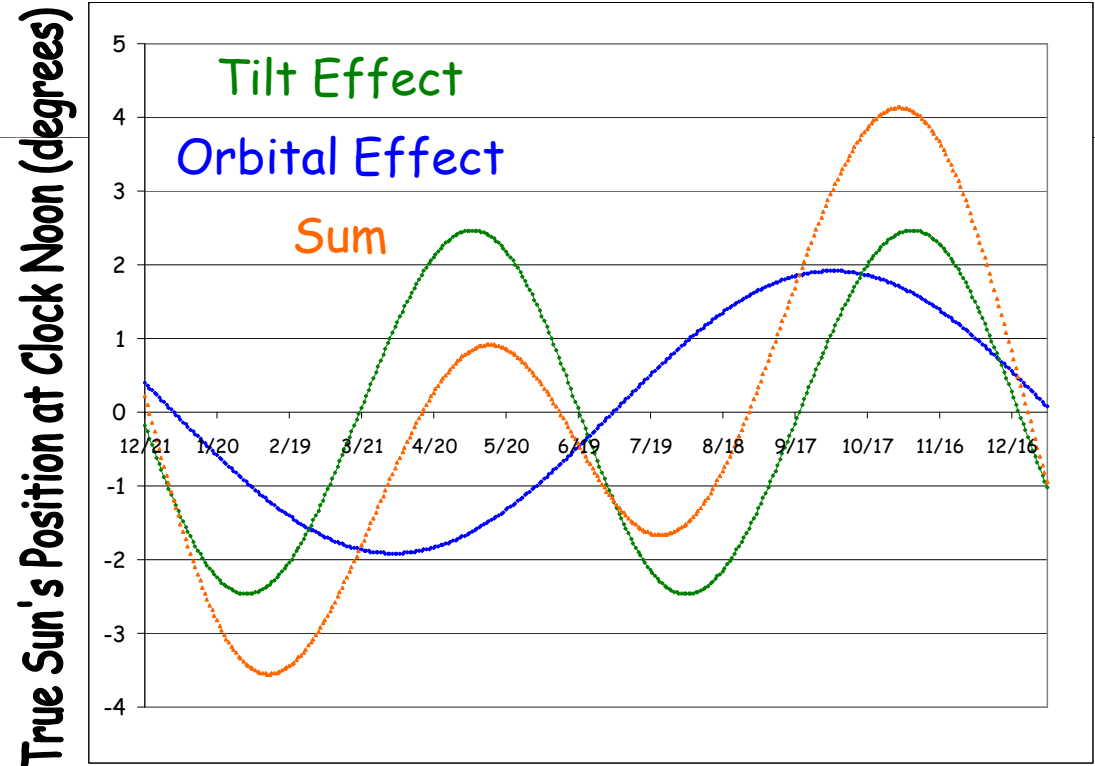
Sun's Position at Clock Noon (Degrees)

# Total Analemma

☆ Tilt and Orbit effects add



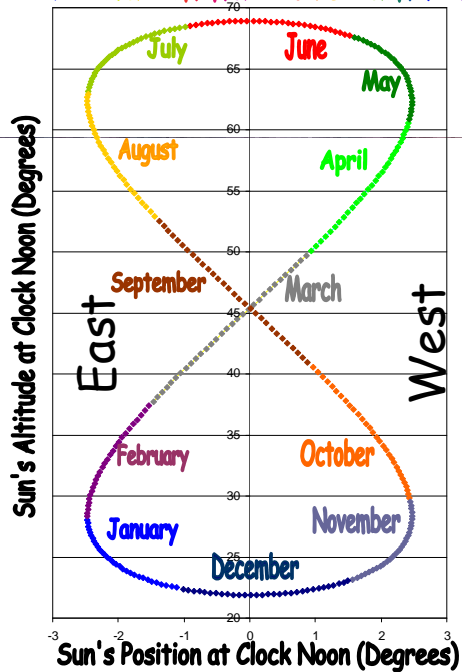
## Equation of Time: Potsdam, NY



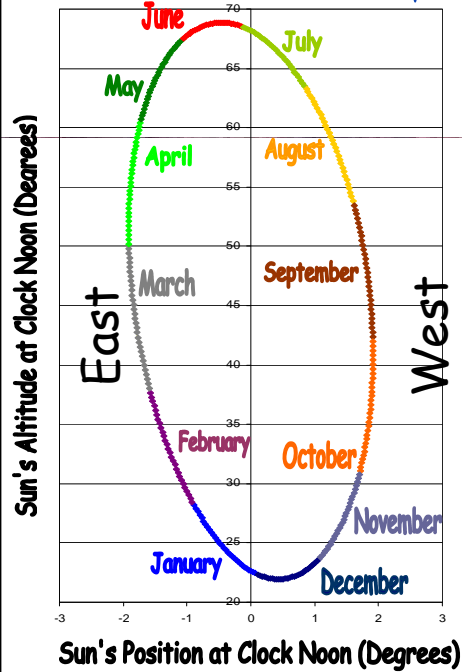
# Total Analemma

☆ Tilt and Orbit effects add

Tilt Analemma: Potsdam NY

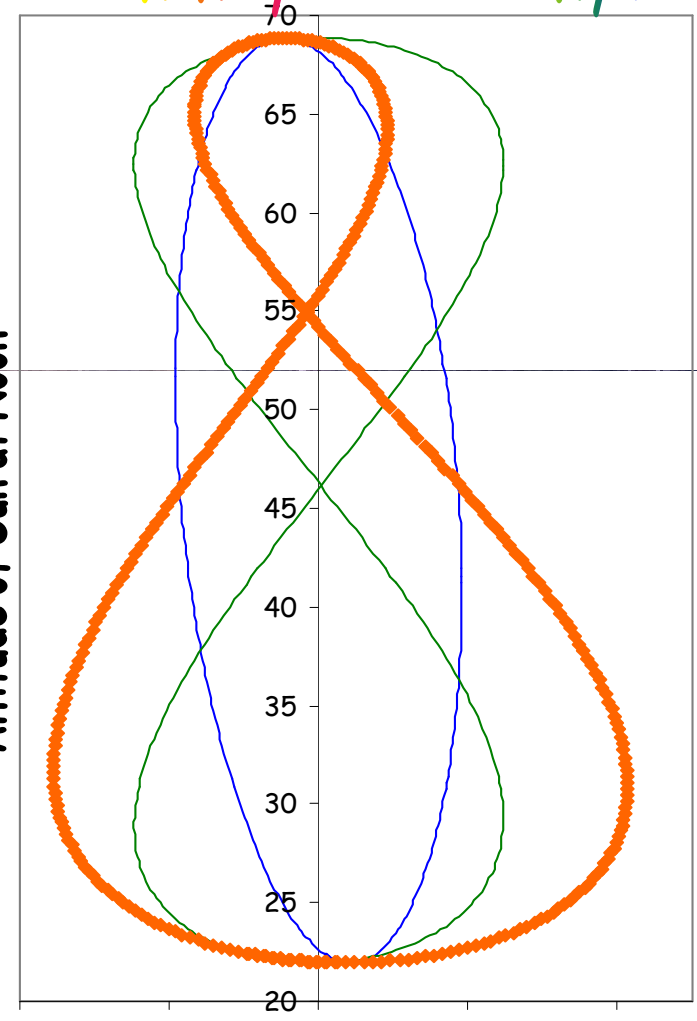


Orbit Analemma: Potsdam, NY



Analemma, Potsdam, NY

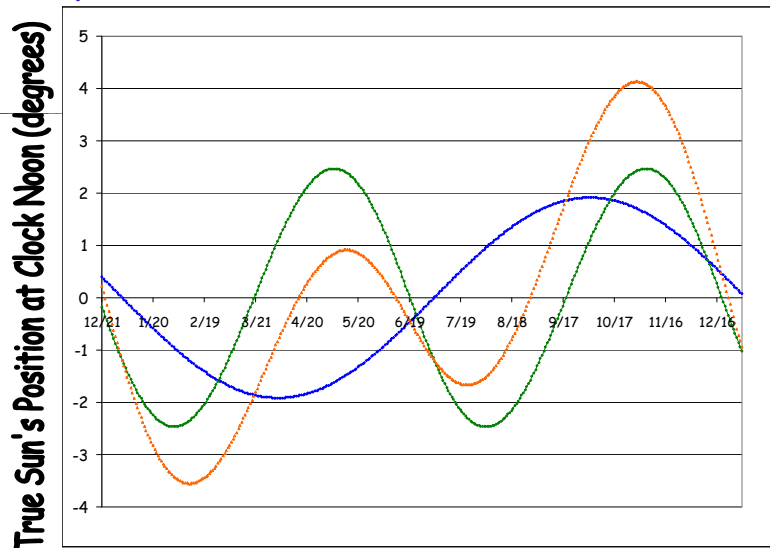
Sun's Altitude at Clock Noon (Degrees)  
Altitude of Sun at Noon



Sun's Position at Clock Noon (Degrees)

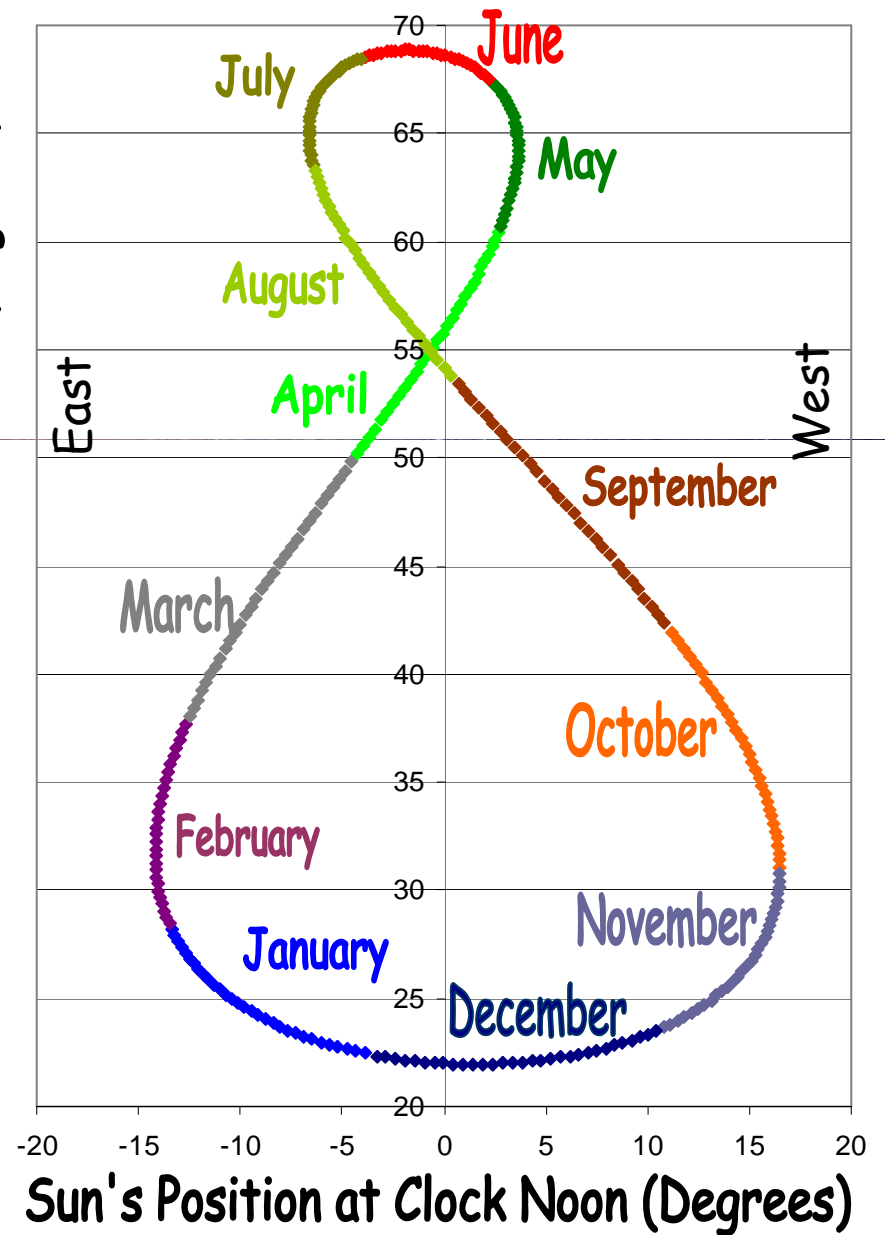
# Potsdam's Analemma

## Equation of Time: Canton, NY



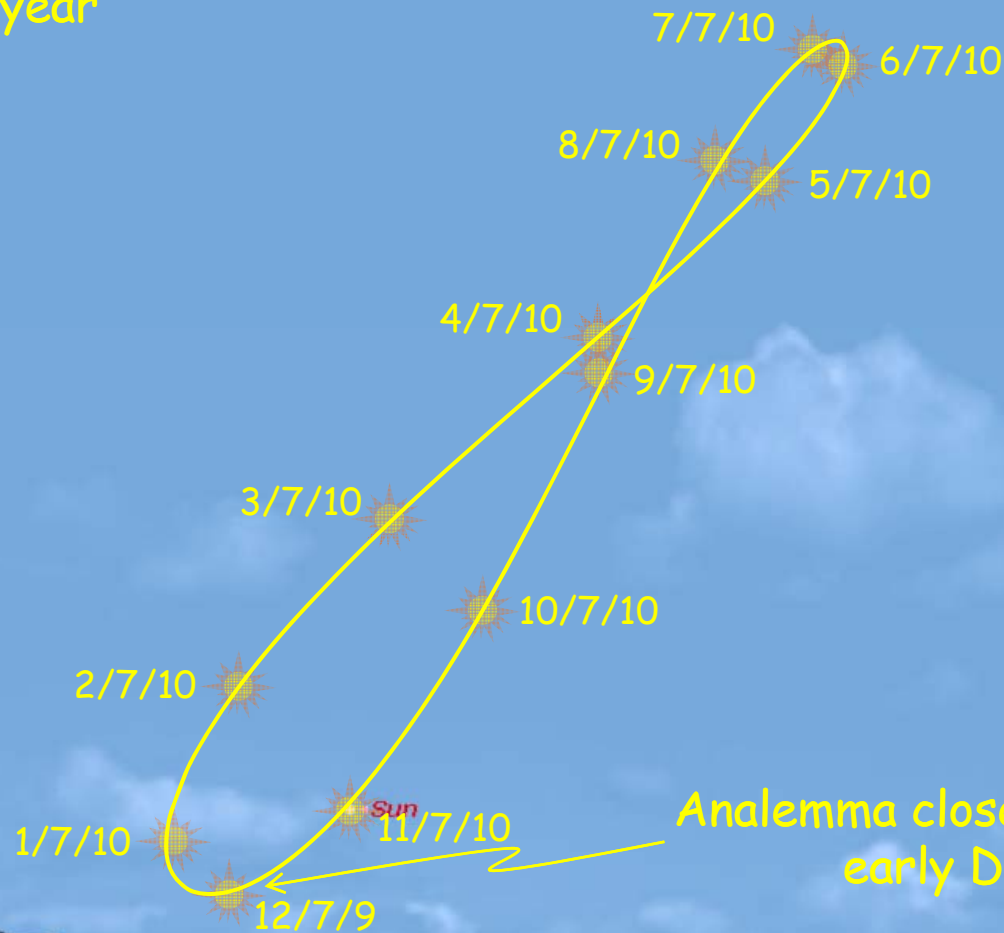
# Analemma, Potsdam, NY

## Sun's Altitude at Clock Noon (Degrees)



# Why is the earliest sunset on December 7, 2009?

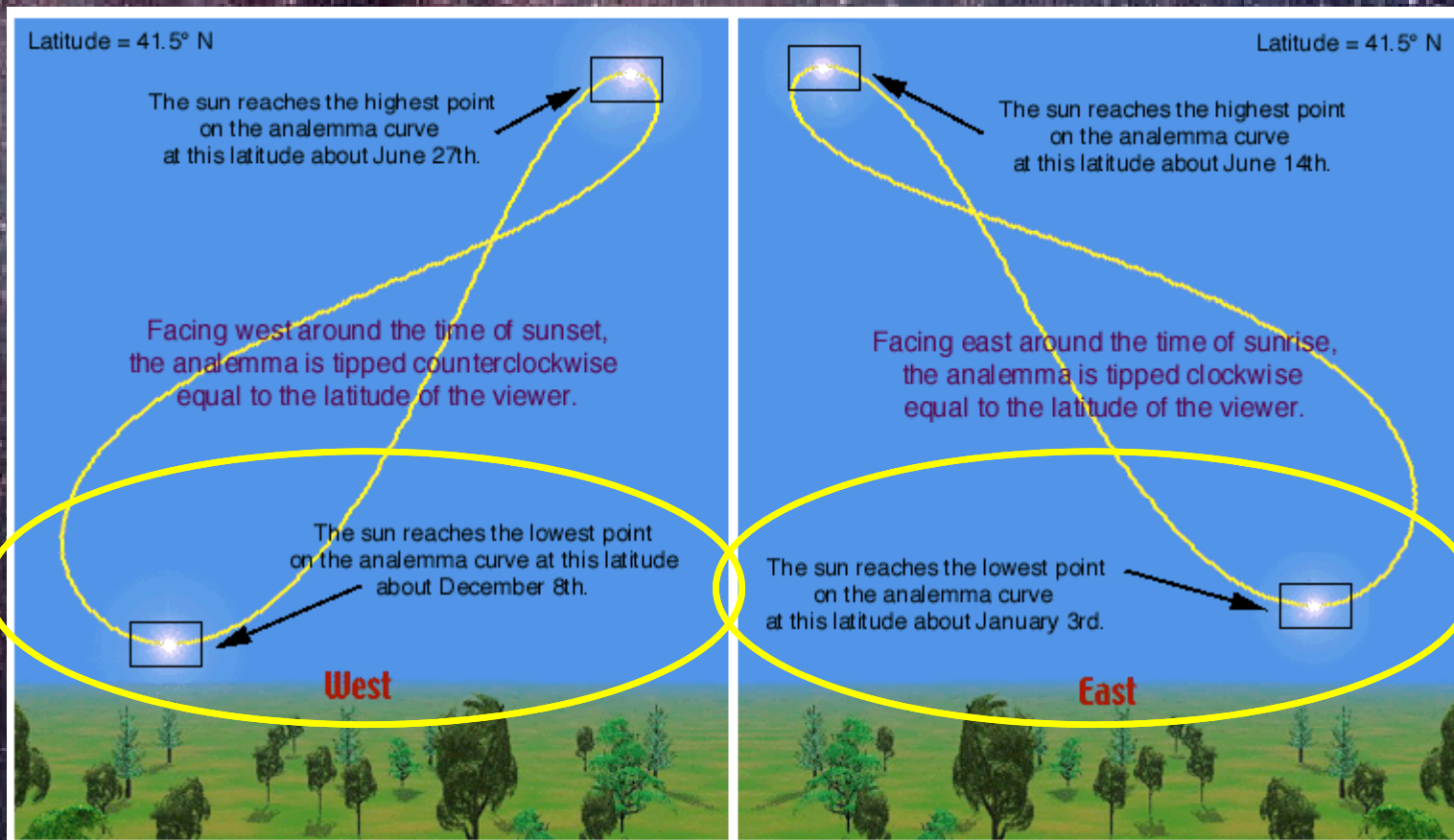
Sun position at 3:30 pm EST  
through the year



Analemma closest to horizon in  
early December

# The Analemma

- ☆ Varies time of sunrise & sunset
- 🌍 Earliest sunset on about December 8
- 🌍 Latest sunrise on about January 3



# The Analemma

☆ Position at clock noon through

At clock noon sun's declination is  $+3^\circ$

clocks

Solar noon to solar noon

